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FEED-FORWARD CONTROL OF TOTAL RETRIEVAL OF THE SPACE TETHER FROM VERTICAL POSITION

This paper presents the results of studies of the feasibility of total and safe retrieval of a space tether of two bodies connected by an elastic, massless cable. The purpose of the research is to build up the control for the mode of the total and safe retrieval of the tether, which is one of the basic modes of its functioning. It allowed the development of the feed-forward control of the tether length or tension that provides demanded change of the angular momentum of the tether under the effect of the gravitational torque. The novelty of the research results consists also in the novel approach to the control of underactuated mechanical systems, which have the number of the control channels less than their degrees of freedom. Here the constraints on the tether angular motion relative to the pitch axis are introduced. They reduce the number of the system degrees of freedom and allow realizing the necessary mode of motion. For this control, only the remaining degree of freedom is used. The numerical simulation of the effect of the mode parameters on the tether motion is carried out for the tether in the chosen ranges of the parameters. The numerical example demonstrates the simplicity of the application of the method in practice. Plots illustrate the analysis of the results.

Keywords: *elastic space tether, underactuated mechanical system, retrieval, feed-forward control.*

Annex Nomenclature

c_i $(i = 0, ..., 7)$	= coefficients of power series
EF	= cable longitudinal rigidity
$e_{ri}(j)$ $(i = 1, 2; j = 1, 2, 3)$	= directing cosines of the position vectors of point masses in the orbital frame of reference
$\mathbf{K}^C(t)$	= vector of angular momentum about mass center C
$K_3^C(t)$	= projection of angular momentum on Cz^{or} axis

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L, L_0, L_f	= current, initial, and interim distance between end point masses
$m_i\ (i = 1, 2)$	= masses of end bodies
$\mathbf{m}^C(t)$	= vector of external moment
$M_g(t)$	= gravitation torque magnitude
$\mathbf{r}_1, \mathbf{r}_2$	= position vectors of the point masses relative to point C
$r(t)$	= position vectors magnitude
$r_p(t)$	= program law of $r(t)$ change
$r_{pe}(t)$	= program law of $r(t)$ change with correction for elasticity
T	= force of cable tension
T_f	= time of increasing value of pitch angle to $\vartheta = \pi / 4$
T_r	= total time of retrieval
t	= time
$S(t)$	= speed of change of length $L(t)$
$x_i^{or}, y_i^{or}, z_i^{or}\ (i = 1, 2)$	= projections of $\mathbf{r}_1, \mathbf{r}_2$ on orbital frame
$\Delta r(t)$	= correction of program law $r_p(t)$ for elasticity of cable
ϑ	= pitch angle
ω^{or}	= angular velocity of orbital tether motion

1. INTRODUCTION

Space tethered systems (STS) attract the extensive attention of a researcher due to their potential applications for space technologies. The volume of publications devoted to the research of various modes of motion of the STS is very large. A comprehensive analysis of potential STS applications is presented in [6, 11]. Problems of deployment and retrieval of a tether of two space bodies take a special place among the problems that are connected with the STS dynamics. A considerable volume of publications is devoted to these problems. They differ by the accepted mechanical model of the tether and the character of the control of tether putting into the final state. An in-depth review of research in this area contains in publications [4, 15—17, 19, 26, 27, 32—34]. Nevertheless, we will dwell here on some issues additionally.

The vertical STS configuration is stable in the orbital frame of reference in the case of the constant length of the cable. Theoretically, it is true for the spherical Earth. One can assume that, within engineering accuracy, this hypothesis is valid [28]. The configuration of the tether aligned to the local vertical loses the stability with the change of its length according to the theorem of the angular momentum change [12]. The requirement that the tether should be on the local vertical during the mode of transformation could be reached only at the expense of enhancement of the control system. The fulfillment of this requirement is not often necessary from the point of view of the practice.

The modes of deployment and retrieval are the basic modes of functioning of space tethers, which are needed for practice. Usually, in publications, researchers have considered simultaneously both modes. However, it must be taken in mind that each mode has its specifics.

A number of authors consider the free deployment at the initial stage [25]. In this case, after initial pushing away of a subsatellite from a mother spacecraft, the tether is deployed practically without resistance under the action of forces of the gravity gradient. As a result, after braking of the tether at the end of the deployment, the tether enters the mode of libratory (pendulous) oscillations in the orbital plane about a vertical equilibrium position. Finally, the tether settles down because of internal damping in the visco-elastic cable in a position along the local vertical. This process lasts very long, and methods of its speedingup have been developed. In [3], the pendulous oscillations are described, which arise with the primary deployment of a tether with a visco-elastic cable, and are damped by parametric pulling up the cable near the point of return of pendulous oscillations and deploying it again in the vicinity of the local vertical. This is the inverse process of parametric excitation of a pendulum. The linear feedback control helps to reach the final equilibrium more effectively as soon as the amplitude of pendulum oscillations becomes sufficiently small.

A controlled tension of a cable at the first stage of the deployment is considered in [4]. The description and analysis of various ways of the deployment of orbital tether systems with control of the deployment speed are described in [6, 10, 11, 13].

All scenarios of deployment of orbital tether systems with the control of the deployment speed have one common shortcoming. This is because, with such a control, it is impossible to execute damping of longitudinal oscillations of the tether, which arise due to its flexibility. Increasing the intensity of longitudinal oscillations can lead the tether to the loss of the tension. That, in turn, can result in the loss of control of its deployment or retrieval.

There are known ways of deployment of the orbital tether systems that provide the deployment of a cable with the regulation of its tension force. The basic advantage of these ways of deployment is the capability of direct damping of longitudinal oscillations of the tether during its deployment. A variety of different control strategies of such ways of deployment and devices for their realization are described in [16, 17, 19]. The time-optimal control problem for a simple system with a massless cable is solved in [24]. As a result, a bang-bang control law is found for the cable tension. It allows moving the subsatellite from a position of relative equilibrium along the local vertical to the same configuration at a significant distance from the mother spacecraft without essential deviation of the subsatellite from the local vertical and with a constantly taut cable. A deficiency of this method is the complexity of the practical realization of the tether tension regulation when the tension force is very small.

In the paper [4], a comprehensive investigation of the controlled deployment comparing six various scenarios is described. Based on the method of multiple shooting, the optimal controllers for the creation of a control force for the STS deployment and retrieval are developed in [23, 24]. The controller for the STS motion in the orbital plane is offered in [29], which shows a high computing efficiency. The paper [31] also compared the various functions of cost for the STS optimal control in the in-plane case. This research has been expanded later to the STS librations' control in elliptical orbits via tracing periodic trajectories of the libration [30].

The so-called "exponential" deployment is one of the most discussed deployment scenarios. This deployment has been adopted by many authors, starting with Eades [9]. As a rule, the exponential deployment is considered by the authors as an additional stage for the initial deployment. Such a deployment is discussed in detail in [6, 18] and in many other publications. It is defined in [6] as the deployment with a velocity that is proportional to the length of the deployed tether. For a circular orbit, the authors consider the law of time variation of the deployed tether length r in the form $r = r_0 \exp(-3/4\omega t \sin 2\vartheta)$, where r_0 is the initial tether length, t is time, ω is the orbital angular velocity, ϑ is the tether pitch angle. The exponential deployment is defined in [18] as the deployment for which the deployed length is a function that grows exponentially with time from a certain value r_0, which cannot be zero. The planar motion of a dumbbell shape with the exponential control of the length has been investigated in [14]. It was found that the tether can be deployed without rotation. When using the exponential law, the velocity of the tether's deployment reaches the maximum value at the end of deployment that leads to a jerk and increase in tension force. It can generate considerable longitudinal oscillations of the tether. Alternatives to the methods discussed here are the methods, which have been developed by Banerjee in Refs. [1, 2, 8].

The mechanical models of tethers in the plane of a circular orbit with the massless cable have only one control channel that may be used for change of the tether length. However, the number of its degrees of freedom equals two. The second degree of freedom corresponds to the tether rotational motion about the pitch axis. At such a statement of the problem, one can consider the tether as an underactuated mechanical system, in which the number of degrees of freedom is more than that of the control channels.

Ignoring this feature of the tether mechanical model with the massless cable led to a lack of the solution acceptable for the practice of deployment of a tether in the vertical position for a long time.

Control of underactuated systems is always a challenging problem. There is a series of studies where the problem of the build-up of controllers for such systems is considered. Here it is possible to note the publications [20] where the control is optimized, [21] in which an input shaping scheme to generate the control signal is offered, [22] where a partial feedback linearization method is used, and others. As regards the research of space tethers within the framework of the theory of underactuated mechanical systems, the authors may note the publication [27] where the tether is considered as an underactuated system. In this publication, the energy approach for controller development is used. Based on the analysis of the dynamics, an ingenious virtual signal is designed, and a control scheme is proposed using the system passivity. With the addressed virtual signal, the coupling behavior between the controllable tether length and uncontrollable in-plane angle is enhanced. Under the scheme, the uncontrollable in-plane angle can be controlled by using the coupled tether length.

One may avoid the difficulties of realization of the scenarios and complexities of the control laws for modes of deployment/retrieval of a tether in a circular orbit, described in [33], if to impose the additional constraints on the system reducing the number of its degrees of freedom. At this, the system ceases to be underactuated one, and the problem of creating the feed-forward control becomes essentially simpler. The constraints' structure should correspond to the parameters of the investigated mode. Its structure must consist of the suitable association of the pitch angle with time. It will be shown below how it works.

To avoid the shortcoming related to control of the speed of a cable or its tension, we will also show how to build up the laws of the cable's length control.

The total retrieval of a tether during one stage is of essential interest for practice. At that time, there should not be dangerous situations when the cable can be reeled on a spacecraft structure. Besides, if there is a safe possibility to carry out the total retrieval of a tether as fast as possible, it can be made without the attraction of the theory of optimum control. For this, the restriction on a tether motion on the pitch angle should provide such trajectories of the end bodies for which velocity of decrease of the angular momentum of the tether will be maximal feasible. Further, we will show how it works.

2. MATHEMATICAL MODEL OF THE SYSTEM

Let us distinguish two basic types of tethers. The tethers consisting of space bodies with essentially different masses can be referred to the first type. For example, a small probe deployed from a large spacecraft. Tethers of two bodies having similar or equal masses can be referred to the second type.

Without loss of generality of the problem formulation, one can choose two equal point masses connected by an elastic massless cable as the tether mechanical model. The neglect of the sizes of the end bodies is well-grounded here because motion modes in which the cable can be reeled on the end bodies are not considered here. The neglect of the cable mass with respect to masses of the end bodies is quite substantiated for tethers with non-electroconductive cable made of modern light materials. This problem has been depicted in several publications, in particular in [12]. The authors have convincingly shown that the motion of tethers with the massive cable described by the differential equations in partial derivatives coincides with the motion of the tether with the massless cable. One may consider also that the center of gravity and mass center of a tether are matched. For example, these points are apart about 1 m when the length of a vertically located tether is equal to 5 km. This distance decreases at the rotation of the tether around the pitch axis. This distance is equal to zero rigorously when the pitch angle is equal to $\pi/2$. Thus, one can consider that the mass center of the tether is in a circular orbit.

Let us introduce the right-handed orbital frame of reference $Cx^{or}y^{or}z^{or}$ [5] for the convenience of a further description of the tether dynamics. In this coordinate frame, the Cx^{or} axis points from the Earth's center along the position vector of the mass center of the tether at its orbital motion, the Cz^{or} axis is normal to the orbital plane, and the Cy^{or} axis supplements this orthogonal triad. Note that the Cx^{or} and Cy^{or} axes correspond to the motion in the orbital plane, and the Cz^{or} axis corresponds to the motion out of this plane.

Let us choose the central Newtonian field of forces as the model of the gravity field. The position vectors $\mathbf{r}_1, \mathbf{r}_2$ of the point masses relative to the point C can be defined by their projections in the orbital coordinate frame:

$$\mathbf{r}_1 = \{x_1^{or}, y_1^{or}, z_1^{or}\}, \ \mathbf{r}_2 = \{x_2^{or}, y_2^{or}, z_2^{or}\}.$$

Let us choose these projections together with their time derivatives in the orbital reference frame as the phase variables of the problems. Then one can define the pitch angle as $\vartheta = \text{atan}2(x_1^{or}, y_1^{or})$, where the index «1» determines the upper body in the initial instant of time. Figure 1 shows the mechanical model of the tether with two equal masses in the orbital frame of reference.

Now, one can describe the dynamic model of the system in the view by Hill-Clohessy-Wiltshire (HCW) [7] equations (1), which govern the motion of end point masses relative to the mass center of the tether. Following the traditional derivation, assuming the orbit eccentricity equal to zero, and taking into account the tether tension, the HCW equations can be written as follows:

$$\frac{d^2\mathbf{r}}{dt^2} = \left\{ 2\omega^{or}\frac{dy_i^{or}}{dt} + 3(\omega^{or})^2 x_i^{or} - Te_{ri}(1)/m_i - \right. \\ -2\omega^{or}\frac{dx_i^{or}}{dt} - Te_{ri}(2)/m_i - \qquad (1) \\ \left. -(\omega^{or})^2 z_i^{or} - Te_{ri}(3)/m_i \right\}, \ (i=1,2).$$

Here $e_{ri}(1), e_{ri}(2), e_{ri}(3)$ $(i=1,2)$ are the directing cosines of the position vectors of point masses in the orbital frame of reference; m_i $(i=1,2)$ are the masses of the end bodies; T is the force of the cable tension; ω^{or} is the angular rate of motion in the circular orbit. For numerical simulation, using this system of twelve differential equations of the first order, one should know the expression for the force of tension T at each instant of time. It will be shown below how to find this force.

3. SCENARIO OF THE TETHER RETRIEVAL

Let us consider further the problem of the total retrieval of the tether of two bodies with the massless elastic cable aligned to the local vertical at the beginning. Now, it is necessary to construct the expression for the constraints applied to the directly uncontrollable variable, which is the pitch angle ϑ. Let us consider the tether length (distance between the centers of the end bodies taking into account elastic deformations) as its control variable. From the physical point of view, the required control should ensure decrease of the angular momentum of the retrieved tether relative to the Cz^{or} axis

$$K_3^C(t) = 2\,m\,r^2\left(\omega^{or} + \frac{d\vartheta}{dt}\right)$$

practically to zero. The theorem on change of the angular momentum in the integral form looks like [12]

$$\mathbf{K}^C(t) = \mathbf{K}^C(t_0) + \int_{t_0}^{t}\mathbf{m}^C(\tau)d\tau. \qquad (2)$$

Expression for the gravitational torque, which determines the angular motion of the tether in the considered case, can be written as

$$\mathbf{m}^C(t) = \left[0, 0, -3\,m\,r^2\left(\omega^{or} + \frac{d\vartheta}{dt}\right)^2 \sin 2\vartheta\right]. \qquad (3)$$

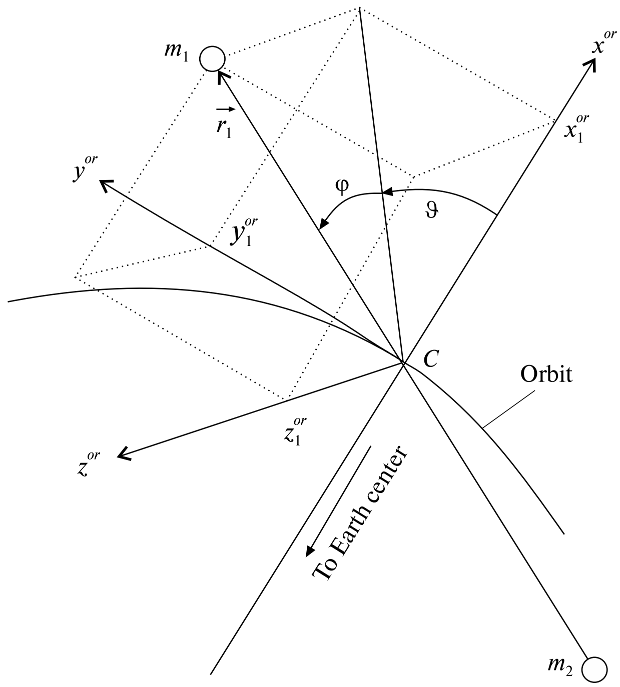


Fig. 1. Mechanical model of tether

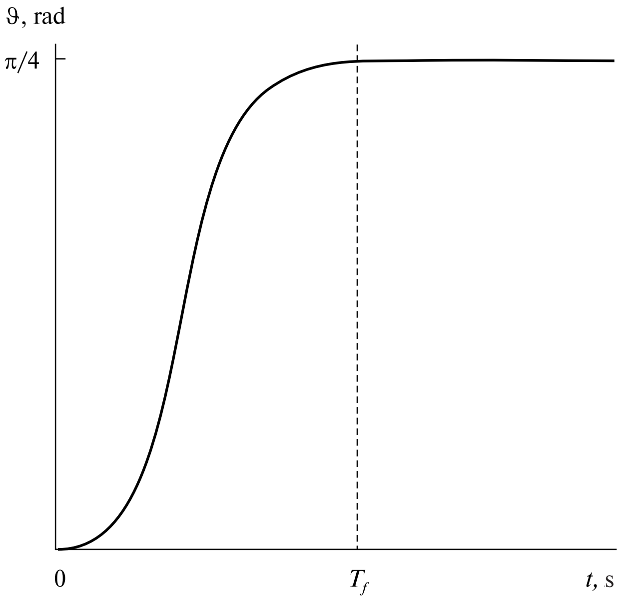


Fig. 2. Crude time history of pitch angle

Obviously that the gravitational torque is maximal at $\vartheta = \pi/4$ at any values r, ω^{or}, and $\dfrac{d\vartheta}{dt}$. Therefore, for the fastest retrieval of the tether, it is logical to choose such a mode of change of the pitch angle, at which the tether appears inclined under angle $\vartheta = \pi/4$ as fast as possible, and the pitch angle remains constant to the end of retrieval. Such a position of the tether also prevents the cable winding on the spacecraft-carrier or rotation of the tether of two bodies of a similar mass at the end of the mode. Let the pitch angle appear on the indicated constant value at $t = T_f$, where t is time in seconds, read out from the beginning of the retrieval mode. Let us consider further the conditions with which the time history of the pitch angle should satisfy, taking into account the previously mentioned information.

As the tether is motionless with regard to the local vertical at the initial instant of time, and the pitch angle becomes constant at the instant of time T_f, the following conditions should be satisfied:

$$\vartheta(0) = 0\,, \quad \vartheta(T_f) = \pi/4\,, \tag{4}$$

$$\left.\frac{d\vartheta}{dt}\right|_{t=0} = 0\,, \quad \left.\frac{d\vartheta}{dt}\right|_{t=T_f} = 0\,. \tag{5}$$

Now, one can imagine the time history of the pitch angle $\vartheta(t)$ crude, as Fig. 2 shows.

Actually, determined part of the $\vartheta(t)$ law is that its part, for which $t \le T_f$. The following additional conditions should be satisfied at retrieval of the tether from the initial length L_0 to its length L_f at the instant of time T_f:

$$L(0) = L_0\,, \quad L(T_f) = L_f\,. \tag{6}$$

The first of these conditions is fulfilled in advance; the second one may be used further for the determination of the unknown in advance instant of time T_f. Its value will allow choosing the suitable solution of the problem from the manifold of solutions obtained for various T_f values.

One more condition follows from the requirement of constancy of the tether length at the initial instant of time:

$$\left.\frac{dL}{dt}\right|_{t=0} = 0\,. \tag{7}$$

The occurrence of jumps of the cable tension is inadmissible both at the terminal instant of the maneuver and throughout all maneuver duration as they can lead to the disappearance of the cable tension. The adopted mechanical model will be inadequate in such a case. The absence of tension jumps at the initial instant of time is reached when the following condition is met:

$$\left.\frac{d^2L}{dt^2}\right|_{t=0} = 0\,. \tag{8}$$

This condition follows directly from the equation of motion of the tether along the cable in the spherical co-ordinates [14] (the case of motion in the orbital plane), which one can write in the symbols adopted here as follows:

$$\frac{d^2L}{dt^2} = L\left[\left(\frac{d\vartheta}{dt} + \omega^{or}\right)^2 + 3(\omega^{or})^2\cos^2\vartheta - (\omega^{or})^2\right] - 2\frac{T}{m}\,. \tag{9}$$

Really, under the conditions (4), (5)

$$\frac{d^2L}{dt^2} = L\,3(\omega^{or})^2 - 2\frac{T}{m} = 0\,. \tag{10}$$

Now it is necessary to build up such a control law of the tether length $L(t)$, which allows to solve the problem in view and completely to retrieve the tether "from rest into rest". To obtain the necessary control law, one can use the theorem on the change of the tether angular momentum. The more simple way is using the equation of the pitch angular motion for a tether of a variable length. Following [14], one can write the equation of the angular motion of the tether in the orbital plane in the following form:

$$\frac{d^2\vartheta}{dt^2}+2\left(\frac{d\vartheta}{dt}+\omega^{or}\right)\frac{dL}{dt}/L+3(\omega^{or})^2\sin\vartheta\cos\vartheta=0. \quad (11)$$

After elementary transformations of the equation (11), one can obtain the ordinary differential equation of the first order with the corresponding initial condition:

$$\frac{dL}{dt}=-L\frac{3(\omega^{or})^2\sin 2\vartheta+2d^2\vartheta/dt^2}{4(\omega^{or}+d\vartheta/dt)}, \; L(0)=L_0. \quad (12)$$

When the law $\vartheta(t)$ is known, the solution of the Cauchy problem (12) has the following form:

$$L(t)=L_0\exp\left[-\int_0^{T_F}\left(\frac{3(\omega^{or})^2\sin(2\vartheta(\tau))+2\,d^2\vartheta(\tau)/d\tau^2}{4(\omega^{or}+d\vartheta(\tau)/d\tau)}\right)d\tau\right]. \quad (13)$$

Using the equation (12) and conditions (4) and (7), one can obtain one more constraint for the pitch angle:

$$\left.\frac{d^2\vartheta}{dt^2}\right|_{t=0}=0. \quad (14)$$

As the pitch angle becomes constant on the range of time $t\geq T_f$, and all its time derivatives vanish, it is possible to consider that the following conditions are satisfied

$$\left.\frac{d^2\vartheta}{dt^2}\right|_{t=T_f}=0,\quad \left.\frac{d^3\vartheta}{dt^3}\right|_{t=T_f}=0. \quad (15)$$

At last, after time differentiation of the equation (12) using already found constraints for $\vartheta(t)$, the following constraint may be added:

$$\left.\frac{d^3\vartheta}{dt^3}\right|_{t=0}=0. \quad (16)$$

Thus, one can use eight conditions (4), (5), (14), (15), and (16) for the construction of the program law $\vartheta(t)$.

One can obtain various laws $L(t)$ of change of the tether length setting various values of the parameter T_f. Then one law may be selected from this manifold of solutions, which corresponds to the set final length of the tether, does not lead to loss of the cable tension during retrieval and meets other introduced requirements.

Necessary law $\vartheta(t)$ may be presented in the form of any finite functional series, and its factors can be found from the above established eight conditions. For example, by analogy with work [33], let us compose the law $\vartheta(t)$ in the form of the power series of seventh degree:

$$\vartheta(t)=\sum_{i=0}^{7}c_i\left(\frac{t}{T_f}\right)^i. \quad (17)$$

Then using eight conditions (4), (5), (14), (15), and (16), one can obtain the system of linear algebraic equations of eight order WRT c_i $(i=0,1,\ldots,7)$, which have the unique solution in the case under consideration. This solution is:

$$\begin{gathered} c_0=0;\quad c_1=0;\quad c_2=0;\quad c_3=0; \\ c_4=35\pi/(4T_f^4);\quad c_5=-21\pi/T_f^5; \\ c_6=35\pi/(2T_f^6);\quad c_7=-5\pi/T_f^7. \end{gathered} \quad (18)$$

The law $L(t)$ obtained according to the expression (13) depends on the law $\vartheta(t)$, which, in turn, depends on the instant of time T_f of increasing the pitch angle to a constant value, duration of the retrieval mode, and the orbital radius. It is essential to note that the obtained law $L(t)$ is always unique under the given form of function and constraints because of the uniqueness of the solution (18).

Hereby, the following contributions of the main problem, which is solved, were made to the development of the scenario of the total tether retrieval:

• the new approach is offered to control build-up for the underactuated mechanical systems, reducing the number of their degrees of freedom and simplifying build-up of the law of control;

• the law of control of tether length is offered and built up;

• the constraints on the tether motion about the pitch axis are formulated, which allow us to make the retrieval mode safe and comprehensible to practice;

• the original trajectory of motion of the tether bodies is offered whose main path segment is located on a straight line (in the orbital basis), inclined at the pitch angle $\pi/4$.

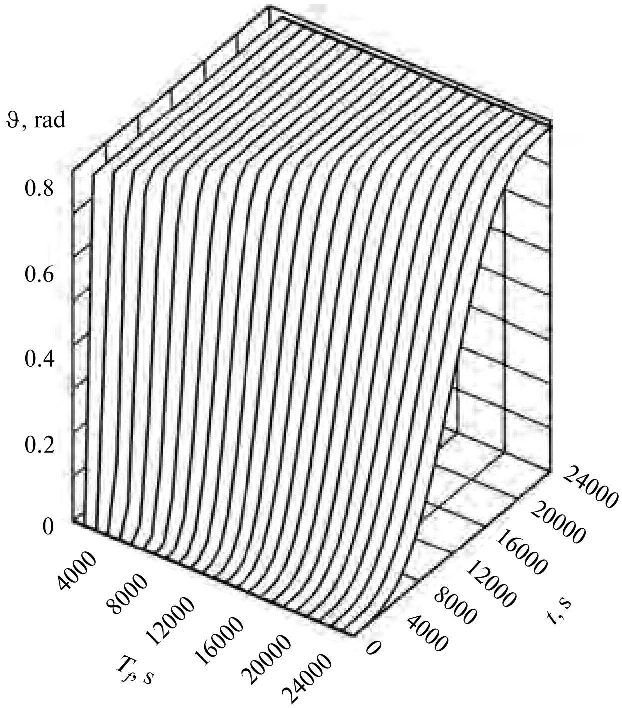


Fig. 3. Pitch angle *vs.* time and parameter T_f

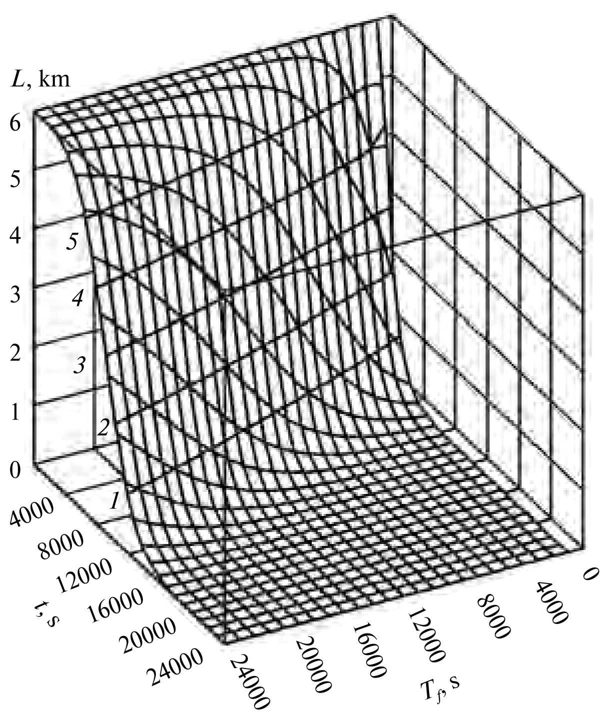


Fig. 4. Length of tether *vs.* time and parameter T_f

• the simple algorithm of the build-up of the control law is stated.

4. NUMERICAL RESEARCH

Let us consider now the practical realization of the proposed method of construction of the feed-forward control for the retrieval mode of a tether, which at the mode beginning is aligned to the local vertical. Let us choose the following values of the tether parameters for the simulation:

Let us show further how to find the law $L(t)$, which solves the problem in view.

Figure 3 shows how the law $\vartheta(t)$ calculated according to the formula (17) depends on various values of the parameter T_f. Each line in this plot corresponds to the separate value of the parameter T_f. The plot shows that the form of the curve $\vartheta(t)$ varies slightly with T_f change, being extended only along the time axis. In order to estimate the acceptability of this or that law of the program motion, it is necessary to build up the program law of the time history of the tether length $L(t)$. One can obtain this law solving the Cauchy problem (12) for a number of values T_f at known law $\vartheta(t)$. As a result, one can obtain the relations shown in Fig. 4. This Figure shows that the length to which the tether may be retrieved at the instant of time $t = T_f$, decreases when the parameter T_f increases. So, at $T_f = 1000$ s, the tether may be retrieved to 4481.01 m, at $T_f = 2000$ s — to 2985.75 m, whereas at $T_f = 24000$ s the tether may be retrieved to 0.12 m.

Nevertheless, the gravitational torque acting on the tether at $t > T_f$ becomes the maximum possible, and the tether retrieval occurs with the maximum possible speed.

As a result, it appears that the faster the tether will be inclined to value the pitch angle equal to $\pi/4$, the faster the tether will be retrieved completely. Figure 4

Table 1. **Tether parameters**

Parameter	Value
Masses of the end bodies are identical	10 kg
Initial length of the tether	6000 m
Longitudinal rigidity of the cable	5000 N
Orbital radius	7000 km

also shows that the shape of the control function $L(t)$ becomes smoother with T_f increase. The isolines on the surface $L = L(T_f, t)$ are marked according to lengths of the retrieved tether from one to five km. The analysis of these isolines shows that one can retrieve the tether to the set length at different values of the parameter T_f, however the less this parameter, the less time required for retrieval.

The dependence of the final length of the tether in meters after the end of the retrieval on the value of the parameter T_f and duration of the retrieval process is shown in Fig. 5. Isolines from one to four meters are also put here. They confirm the conclusion based on the analysis of isolines in Fig. 4.

The speed of change of the length $L(t)$ *vs.* the parameter T_f value and duration of the mode of retrieval is shown in Fig. 6. Here one may see that the speed $V(t)$ at the chosen parameters of the tether can be sign-variable when the end body trajectory goes into the line $\vartheta(t) = \pi/4$ at small values of the parameter T_f. It complicates the program control law essentially as there are time intervals in which the length of the tether temporarily increases. The control becomes complicated, but the results of several last trajectories practically do not differ. Thus, the increase in the parameter T_f can be alternative to the control law complication. The areas like the one shown in the right forward corner in Fig. 6 are of little use for the construction of the programmed control for the mode of the tether retrieval.

The behavior of trajectories of the tether end bodies at the duration of the retrieval $T_r = 24000$ s and various values of the parameter T_f is shown in Fig. 7. The trajectories of the upper end body of the symmetric tether are shown here in the orbital frame. One can see in this plot how the shape of the trajectory changes when the parameter T_f increases. The trajectory of the end body becomes more complex with T_f growth, and there are time intervals where the length of the tether increases temporarily. The control becomes more complex at that, but results of retrieval along a few last trajectories practically do not differ. Hence, one can draw the conclusion that the alternative to the control law complication is the increase of the maneuver duration.

This trajectory becomes either very difficult for realization because of the necessity of switching the

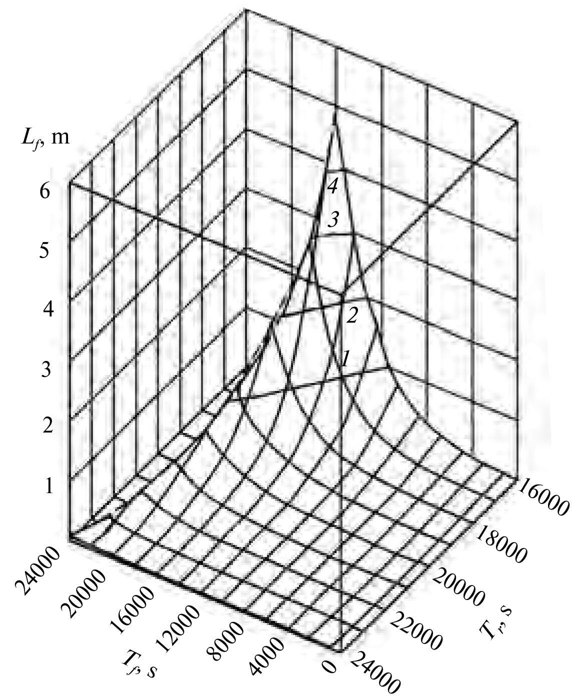


Fig. 5. Final length of tether *vs.* duration of retrieval T_r and parameter T_f

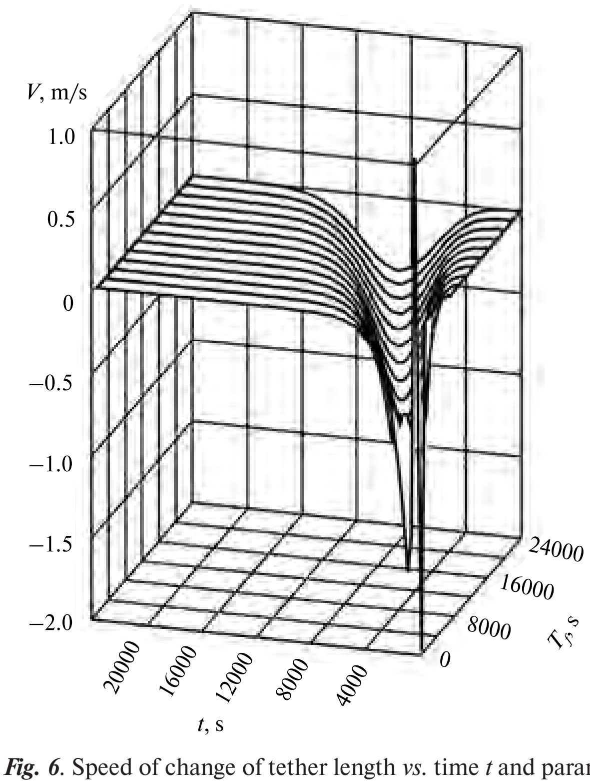
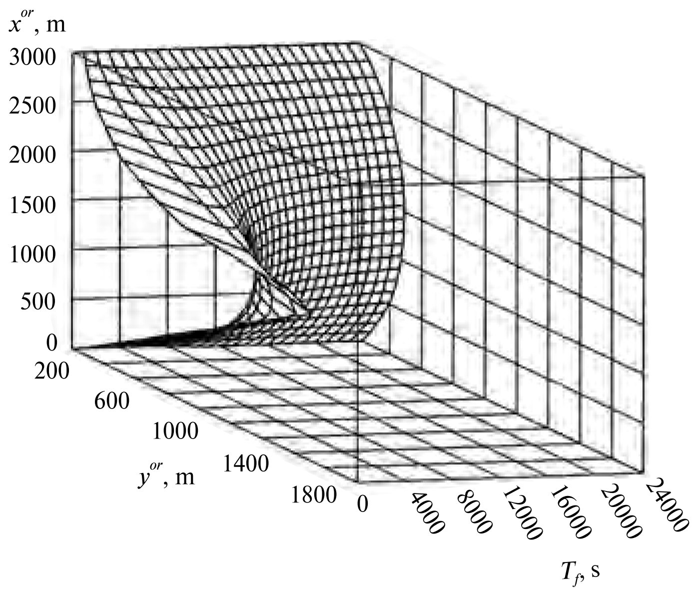


Fig. 6. Speed of change of tether length *vs.* time *t* and parameter T_f



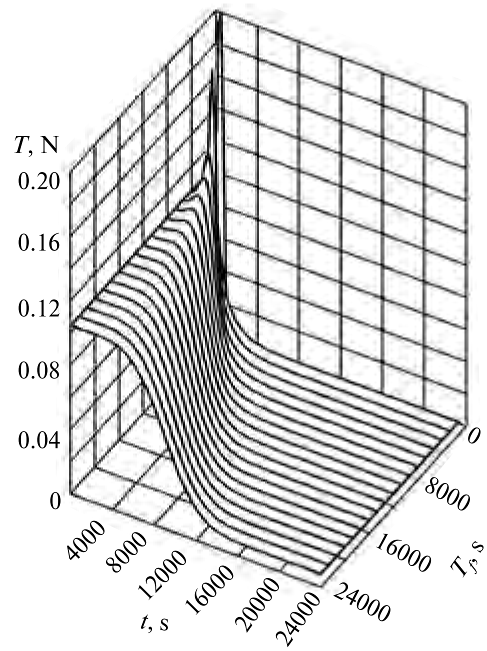


Fig. 7. Trajectories of end bodies at the duration of retrieval $T_r = 24000$ s and various values of the parameter T_f

Fig. 8. Tension force *vs.* time *t* and parameter T_f

speed sign or not realized at values of the parameter T_f smaller than 1500 s. The trajectory of the end body becomes smoother when T_f increases more than 1500 s. Thus, as it has been told, the increase of duration of the maneuver can be the alternative to the control law complication.

An indispensable condition of the realizability of the maneuver of the tether under consideration is the tension of the cable during the entire time of its retrieval. One can find the dependence of the tension force in the cable on time using Eq. (9) when the law of change of the tether length $L(t)$ is known.

The dependence of the tension force in the cable on time is shown in Fig. 8 at various values of the parameter T_f. At $t = 0$, the force of tension is determined from the condition (10). The cable tension is increased at the beginning of the process of retrieval because of overcoming forces of inertia of the end bodies. Naturally, the intensity of this increase is determined by the value of the parameter T_f — the less this parameter, the faster the tether tilts, and, hence, the faster its length decreases. Thus, the behavior of the tension force of the cable becomes essentially more complex in those ranges of the parameter T_f values, where there are time intervals, at which the cable is extended. Thus, the tension decreases at first, when the cable is let out, and then sharply increases when the cable starts to shorten over again. The curve $T(t)$ shown in the foreground corresponds to the force of the cable tension during retrieval when the parameter T_f is equal to 24000 s.

Let us consider further the tether behavior at its total retrieval.

If one chooses the co-ordinates of the end bodies and their first time derivatives in the orbital frame of reference as phase variables of the problem, one can introduce the program force of tension $T(t)$ into the model as the control, which can be calculated on each step of integration, using the equation (9). Program values $L(t)$ can be obtained on each step of integration of the Cauchy problem for equations (12). The first time derivative of $L(t)$ can be determined from (12) after substitution there the expressions for $\vartheta(t)$ and its time derivatives. As a result, one can obtain the expression for tension force $T(t)$, which can be used at numerical integration of the Cauchy problem for HCW equations with help of the computing program written in the FORTRAN. The HCW equa-

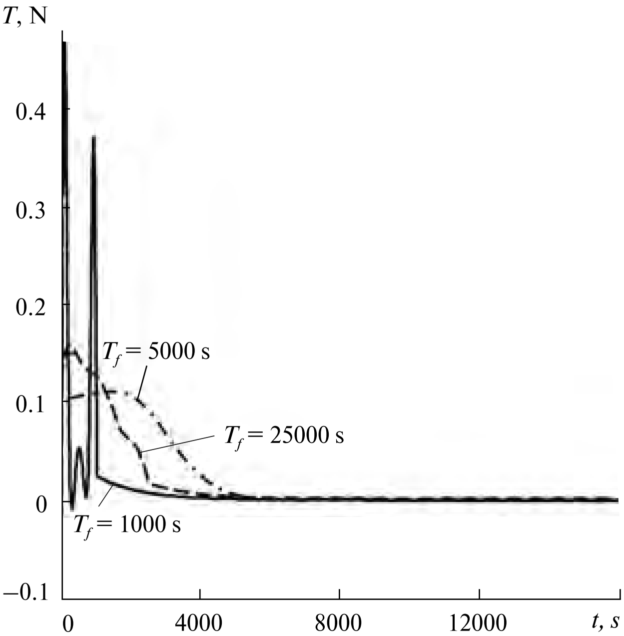


Fig. 9. Tension force $T(t)$ *vs.* parameter T_f and time t at duration $T_r = 16000$ s

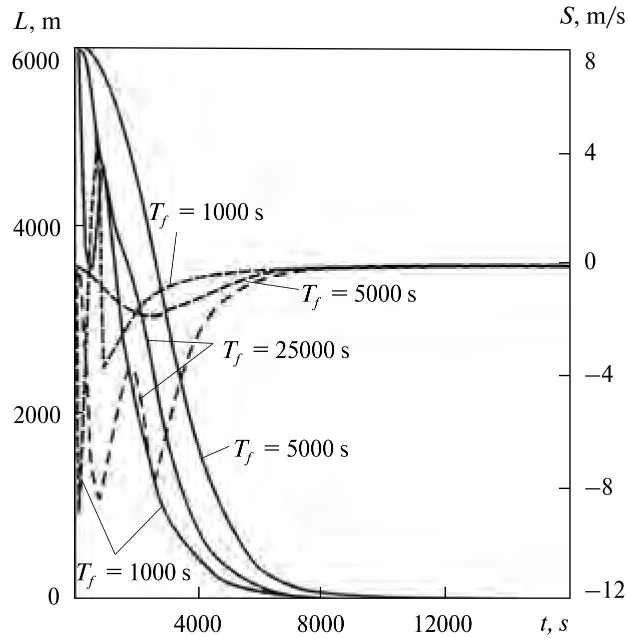


Fig. 10. Tether length $L(t)$ and speed of its change $S(t)$ *vs.* time t

tions and Eq. (12) were programmed for the numerical solution with a Runge — Kutta integration routine of fifth order with a constant step of integration equal to 0.1 s. Using steps of integration 0.2 s and 0.05 s has resulted in the divergence of $L(T_r)$ values about 2—3 mm.

The behavior of the tension force $T(t)$ for three various values of the parameter T_f is shown in Fig. 9 for the case when the duration of retrieval $T_r = 16000$ s. Here one may see that for $T_f = 1000$ s at the time interval close to the initial instant (from t = 260 s to t = 320 s), the tether tension becomes negative, indicating the inadequacy of the adopted model of the tether for such a value of the parameter T_f and all its smaller values. (Calculations have shown that already at $T_f = 1030$ s, such intervals in the case under consideration are absent). The time history of change of the tension force $T(t)$ resulted in the plots for two other values of the parameter T_f (they are shown by dotted and stroke-dotted curves) has smooth enough character and indicates adequacy of the adopted mechanical model.

The time history of the distance $L(t)$ between the end bodies and the speed of change of this distance $S(t)$ is shown in Fig. 10 also for three various values of the parameter T_f at the duration of the retrieval $T_r = 16000$ s. The curves $L(t)$ are shown by solid lines, curves $S(t)$ — by dotted ones. One may see here how the time histories of these values essentially vary as far as the parameter T_f increases. The occurrence of the range of time where the speed $S(t) > 0$ is visible at $T_f = 1030$ s, i.e., the cable is unreeled here. Such ranges are absent at $T_f = 2500$ s, the curve $L(t)$ has a smooth shape, though the curve $S(t)$ is difficult enough for the realization. The maximum value of the speed of winding of the cable does not surpass 4 m/s. Both curves become smooth at $T_f = 5000$ s. The maximal speed of winding of the cable does not surpass 1.63 m/s. Thus, a choice of the value of the parameter T_f proper for realization of the programmed control is defined by requirements to the duration of the mode and by the parameters of the system realizing the control law. Figure 11 shows the trajectories of the tether end bodies in the orbital frame for three different values of the parameter T_f, which defines the instant of entry of the trajectory on a straight line $x^{or} = y^{or}$. The trajectories differ in the shape and complexity of realization, as described in the comments to Fig. 9. The fastest retrieval occurs along the trajectory, which reaches the line $x^{or} = y^{or}$ at $T_f = 1030$ s.

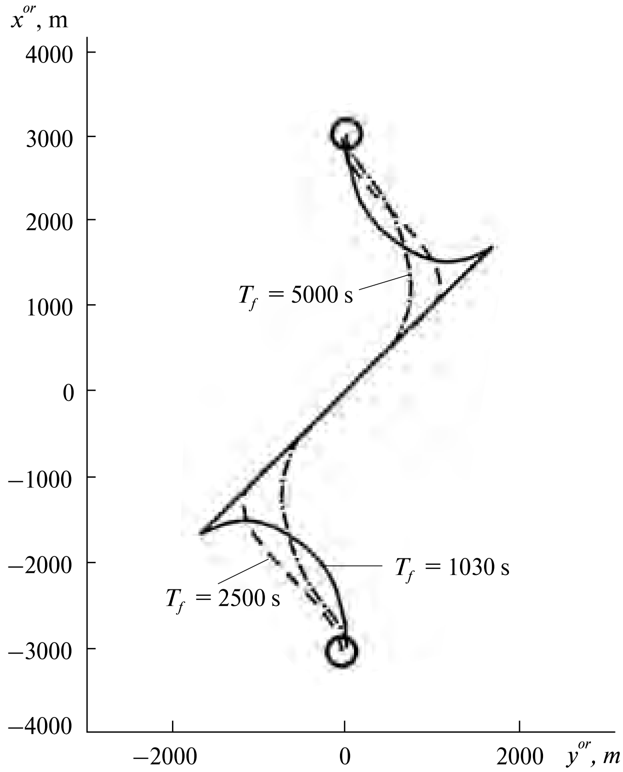


Fig. 11. Trajectories of end bodies in orbital frame

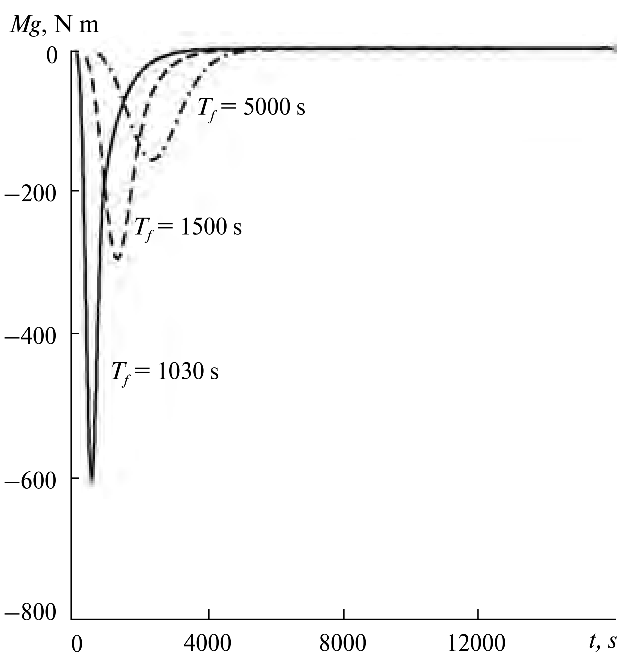


Fig. 12. Time history of gravitational torque

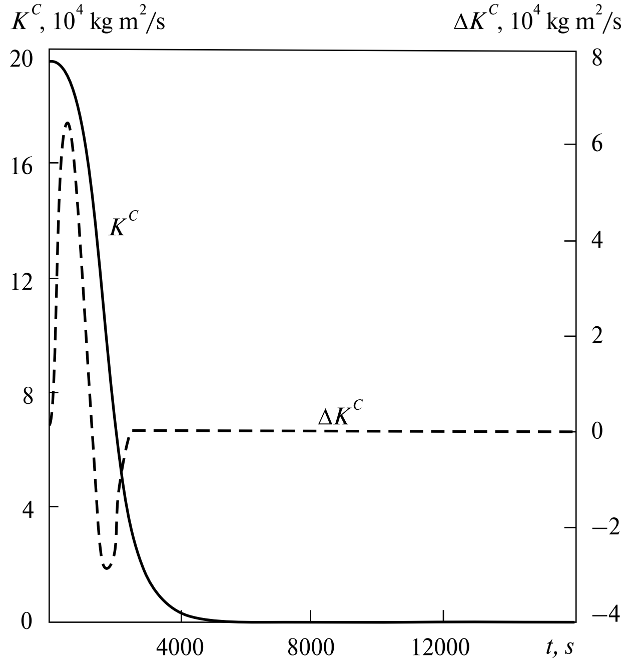


Fig. 13. Magnitude of angular momentum vector of tether and errors of integration

Figure 12 shows the time history of the gravitational torque acting on the tether in the mode of retrieval (see Eq. (3)) for different values of the parameter T_f .

These plots illustrate the fact that the earlier the pitch angle ϑ becomes equal to $\pi/4$, the more rapidly the angular momentum of the tether decreases, and, hence, the more promptly the tether retrieves. Really, the areas of the curves $M_g(t)$, which correspond to the integral in the right part of the Eq. (2), are equal numerically to the total tether angular momentum just before the beginning of retrieval. At time integration, $M_g(t)$ grows the more rapidly, the less value of the parameter T_f for the trajectory.

The stepwise refinement has been introduced into the computing program for taking into consideration the elasticity of the cable when calculating the tension force. Really, the distance from point C to the end body for the flexible cable is:

$$r(t) = r_p(t) + \Delta r(t).$$

Here, $r_p(t)$ is the program law of $r(t)$ change. In turn, $\Delta r(t) = T\, r_p / EF$ is the cable elongation according to Hook's law. Therefore, the quasistatic character of the retrieval mode has been taken into account at calculation T, and the updating has been introduced into the calculation of the program law $r_{pe}(t) = r_p(t) - \Delta r(t)$. The value of $\Delta r(t)$ calculated at the previous step of integration was used as its current value. The step of integration is equal to 0.1 s.

The numerical integration of the Cauchy problem was performed under the monitoring of compliance of the output with the theorem on the change of the tether angular momentum. The magnitude of the angular momentum vector was calculated in two ways. The current values of the phase variables of the initial value problem (1) were used for its evaluation in the first case. In the second one, it was calculated on the basis of the theorem on the change of the tether angular momentum (Eq. (2)). Computational results are shown in Fig. 13. Here the magnitude of the angular momentum vector, calculated using the phase variables values, is shown by the solid line, difference of its evaluation using (2) is shown by the dotted line. Obviously, such errors of evaluations are negligible for practice.

5. CONCLUSIONS

Summing up, it is possible to say that the new approach to the solution of the problem of total and safe retrieval of the vertically aligned elastic space tether of two bodies in a circular orbit is advanced, theoretically proved, and checked up numerically here. This approach is based on the program control of the tether length with the use of the gravitational torque for the tether pitch angle control. From the physical point of view, the purposeful change of the angular momentum of the tether is proposed at the expense of its interaction with the gravitational field, allowing satisfying a lot of additional requirements to the quality of the dynamic processes in the elastic space system. From the point of view of the control theory, the new approach to the control construction is advanced for the underactuated mechanical systems, which have the number of the control channels less than that of the degrees of freedom. Here it is offered to impose constraints on the system motion about the pitch axis, which allows realizing the set mode of motion at control only of the remaining degree of freedom reducing the number of the degrees of freedom of the system. The character of the constraints for the admissible law of change of the pitch angle with time is defined by the requirements shown to the executed mode of formation of the programmed control. These requirements allow retrieving of the tether of two bodies without excitation of longitudinal oscillations because the obtained control law is very smooth *vs.* time and does not contain features, which can call such oscillations. The led example of the application of the stated approach to the case of retrieval of the particular tether demonstrates the simplicity of use of this method in practice.

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Conflict of interest Statement

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence the work. There is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of the work. On behalf of all authors, the corresponding author states that there is no conflict of interest.

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ПРОГРАМНЕ КЕРУВАННЯ ПОВНИМ ЗГОРТАННЯМ КОСМІЧНОЇ ЗВ'ЯЗКИ З ВЕРТИКАЛЬНОГО ПОЛОЖЕННЯ

Представлено результати досліджень повного та безпечного згортання космічної зв'язки двох тіл, з'єднаних пружним невагомим тросом. Мета дослідження полягає у побудові програмного керування для режиму повного та безпечного згортання зв'язки, який є одним з основних режимів її функціонування. Це дозволяє створити закон керування довжиною або натягом зв'язки, який забезпечує необхідну зміну кінетичного моменту зв'язки під дією гравітаційного моменту. Новизна результатів дослідження полягає також у новому підході до керування малоприводними механічними системами, у яких кількість каналів керування менша за кількість ступенів свободи. Введено обмеження на кутовий рух зв'язки щодо осі тангажу. Вони скорочують кількість ступенів свободи системи та дозволяють реалізувати необхідний спосіб руху. Для такого керування використовується тільки ступінь свободи, що залишається. Чисельне моделювання впливу параметрів режиму на динаміку зв'язки виконано для обраних діапазонів параметрів. Чисельний приклад демонструє простоту застосування методу на практиці.

Ключові слова: пружна космічна зв'язка, малоприводна механічна система, згортання, програмне керування.