https://doi.org/10.15407/knit2023.04.003
UDC 629.7

Yu. M. GOLDSHTEIN, Senior Researcher, Cand. Sci. in Tech.
ORCID.org/0000-0002-3931-2680
E -mail: jura_gold@meta.ua

Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and State Space Agency of Ukraine
15, Leshko-Popel Str., Dnipro, 49005 Ukraine

ORBITAL STRUCTURE OPTIMIZATION TECHNIQUE OF THE LOW-ORBIT COMPLEX OF ON-ORBIT SERVICE

Most of the currently planned on-orbit servicing (OOS) missions involve the use of disposable OOS spacecraft. The use of disposable OOS spacecraft may be profitable in the near future. But it is not a reliable solution for OOS in the long term. As an alternative, a more useful concept is the use of reusable OOS complexes, which allow responding to scheduled and random requests from OOS clients. This concept can ensure the timeliness and efficiency of OOS implementation during planned services and random requests of OOS clients. However, despite the potential advantage of a reusable OOS, the design of its orbital structure and operational maintenance is much more complicated in comparison with the traditional concept of the organization of OOS. This is because when planning the response of reusable OOS complexes to requests, it is necessary to distribute OOS client service operations between space vehicles of the reusable OOS complex. Now the space industry is switching its attention to the area of low Earth orbits. This causes an increase in deployed and planned low-orbit satellite groups, the number of satellites in them, the difference in structural schemes of satellite groups, and the significant influence of the environment on orbital parameters. As you know, the orbital parameters of low orbits of space vehicles can differ significantly, and the difference between them can reach tens or even hundreds of degrees in the longitude of the ascending node. This leads to unacceptably high energy costs for modern OOS spacecraft for active rotation of the planes of their original orbits to the planes of the destination orbits. In some works, the possibility of reducing these energy costs due to the use of the difference in the speed of the nodal precession of the parking and destination orbits of the OOS spacecraft due to the non-centrality of the Earth's gravitational field is considered. However, due to the long wait of the OOS spacecraft in the parking orbit, the flight time with the wait between the parking and destination orbits increases significantly. Its reduction can be achieved by increasing the number and rational selection of the semi-major axis and inclination of the parking orbits of the OOS spacecraft. The purpose of the article is to develop a technique for the optimal synthesis of the orbital structure and optimal operational planning of the low-orbit OOS complex in near-Earth orbits with a small eccentricity. Methods for solving the problem are the averaging method, the branch-and-bound method, and the multi-objective optimization method. The novelty of the obtained results lies in the development of a technique for optimal synthesis of the orbital structure and optimal operational planning of the low-orbit space OOS complex in near-Earth orbits with low eccentricity. The developed technique can be used in the previous planning and design of space OOS complexes in low near-Earth orbits with a small eccentricity.

Keywords: *multi-objective optimization, parking orbit, Pareto front, on-orbit service, low thrust, averaging method.*

Цитування: Goldshtein Yu. M. Orbital structure optimization technique of the low-orbit complex of on-orbit service**.** *Space Science and Technology.* 2023. **29,** № 4 (143). P. 3—11. https://doi.org/10.15407/knit2023.04.003

1. INTRODUCTION

At the current stage of the development of space activities, the world's leading space states are actively developing promising on-orbit services (OOS). OOS is turning into a viable space industry, the development of which is determined by the development of new space technologies, which include OOS industrialization and the development of modular spacecraft. It is expected that OOS industrialization and modular satellites will be revolutionary technologies for the satellite industry [7, 11, 12].

Most of the OOS missions that are planned and executed at this time use disposable OOS spacecraft. The use of disposable OOS spacecraft may be profitable in the near future. But it is not a reliable solution for OOS in the long term. As an alternative, a more useful concept is the concept of industrialization of OOS due to the deployment of reusable and sustainable OOS complexes [11, 12].This concept can ensure the timeliness and efficiency of OOS implementation for scheduled and random requests of OOS clients for service. However, despite the potential advantage of using reusable OOS complexes, the design of their orbital structure and operational service planning is much more complex and difficult compared to the traditional concept of OOS organization. This is because when planning the response of the OOS complex to the requests of OOS clients, it is necessary to carry out the distribution of OOS client service operations between spacecrafts of the OOS complex. According to experts, for the effective functioning of the OOS complex, the number of OOS spacecrafts in their structure can vary from several units to tens.

In modern literature, the main attention is given to certain aspects of OOS missions performed by disposable OOS spacecraft. In [16], the problem of refueling multi-purpose spacecraft in near-Earth orbit was studied. The effect of perturbing the orbit due to the non-sphericity of the Earth is also considered. Papers [1, 2, 13] studied the problem of space debris removal. In work [14], using the discrete particle swarm method, the planning of a mission to serve several OOS clients was studied. Refueling strategies using OOS spacecraft with low-thrust engines were investigated in [5].

Only a relatively small number of works are devoted to the development of methods for optimal planning of the orbital structure and operational planning of low-orbit reusable OOS complexes [3, 4]. In this regard, there is currently a need to improve the existing and develop new methods for solving these problems.

The purpose of the article is to develop a technique for the optimal synthesis of the orbital structure and optimal operational planning of the low-orbit OOS complex in near-Earth orbits with a small eccentricity. Methods for solving the problem are the averaging method, the branch-and-bound method, and the multi-objective optimization method. The novelty of the obtained results lies in the development of a technique for the optimal synthesis of the orbital structure and optimal operational planning of the low-orbit OOS complex in near-Earth orbits with a small eccentricity. The developed technique can be used in the justification, planning, and design of OOS complexes in low Earth orbits with a small eccentricity.

2. FORMULATION OF THE PROBLEM

It is assumed that the OOS complex, consisting of n reusable OOS spacecrafts with engines of low constant thrust, has been deployed. It is designed to fulfill scheduled or random service requests from m OOS clients. In the problem, it is assumed that the base orbits of the OOS spacecraft have a small eccentricity, and their orbital planes have the same speed of nodal precession. To ensure synchronization of nodal precession, their orbits have the same semi-major axes and inclinations. The planes of the base orbits of the OOS spacecraft are evenly distributed along the longitudes of the ascending nodes (LAN). One OOS spacecraft is located in each orbital plane. It is assumed that the orbits of OOS clients also have a small eccentricity and differ little in inclination between themselves and the orbits of OOS spacecraft. In the problem, a significant difference in the orbits of the OOS clients along the major semi-axes and LAN is allowed. The difference in the LAN precession speeds of the OOS spacecraft and OOS clients is achieved due to the difference in the semi-major axes of their orbits. It is assumed that interorbital flights are the main factor affecting fuel consumption and the time of execution of the OOS. Any of the OOS spacecrafts can perform any of the OOS execution requests. Each OOS spacecraft can be assigned to perform only one OOS execution request, and each

OOS, in turn, must be performed by only one OOS spacecraft. The work does not take into account the effect of gravitational disturbances of a higher order than those associated with the influence of the second zonal harmonic of the Earth's geopotential. Disturbances due to the influence of the Earth's atmosphere and the gravitation of the Moon and the Sun are not taken into account.

As a result of solving the considered problem, it is necessary to develop methods for the rational synthesis of the orbital structure and optimal operational planning for the low-orbit OOS complex in near-Earth orbits with a small eccentricity. The technique is designed to determine in an acceptable time: the parameters of interorbital flight of OOS spacecrafts, optimal waiting times of OOS spacecrafts on parking orbits, determination of the appropriate distribution of client service requests of OOS between the OOS spacecraft, determination of the appropriate number of OOS spacecrafts and optimal semi-major axis and inclination of their parking orbits.

3. DETERMINATION OF FLIGHT PARAMETERS OF THE OOS SPACECRAFT

According to the conditions of the problem, parking and destination orbits of the OOS spacecraft have, respectively, the following values of the semi-major axis, inclination, and LAN:

$$a^p, i^p, \Omega_i^p \left(\forall i \in \{1, 2, .., n\}\right);$$

$$a_j^d, i_j^d, \Omega_j^d \left(\forall j \in \{1, 2, .., m\}\right).$$

On the flight orbit of the *i*-th OOS spacecraft from the parking orbit to the *j*-th destination orbit, the flight control of the OOS spacecraft is performed by the engine of low constant thrust at a zero pitch angle due to a change in the magnitude and direction of the thrust yaw angle β_{ij}. The yaw control angle changes its sign every half-turn of the orbit at values of the latitude argument u equal to $\frac{\pi}{2}$ and $\frac{3}{2}\pi$ [6, 8, 15]. The yaw control angle β_{ij} is determined by the expression (1):

$$\beta_{ij} = \begin{cases} -\tilde{\beta}_{ij} & u \in \left[\frac{\pi}{2}, \frac{3}{2}\pi\right], \\ \tilde{\beta}_{ij} & u \in \left[0, \frac{\pi}{2}\right] \cup \left[\frac{3}{2}\pi, 2\pi\right], \end{cases} \tag{1}$$

where $\tilde{\beta}_{ij} \in [-\pi, \pi]$.

For short flights of the OOS spacecraft with engines of low constant thrust, mass reduction due to fuel outflow can usually be neglected. The acceleration ε_i of the *i*-th OOS spacecraft due to engine thrust is constant and has the form (2):

$$\varepsilon_i = \frac{T_i}{m_i}, \tag{2}$$

where T_i and m_i are, respectively, the thrust of the engine and the mass of the *i*-th OOS spacecraft.

On the assumption of zero eccentricity of the flight orbit and using [6, 8, 15], we give the system (3)—(5) averaged by the latitude argument flight equations of the OOS spacecraft from the parking orbit to the destination orbit. This system of equations takes into account the influence of the second zonal harmonic of the Earth's geopotential.

$$\frac{da_{ij}^f}{dt} = 2\sqrt{\frac{(a_{ij}^f)^3}{\mu}}\varepsilon_i \cos\tilde{\beta}_{ij}, \tag{3}$$

$$\frac{di_{ij}^f}{dt} = \frac{2}{\pi}\sqrt{\frac{a_{ij}^f}{\mu}}\varepsilon_i \sin\tilde{\beta}_{ij}, \tag{4}$$

$$\frac{d\Omega_{ij}^f}{dt} = -\frac{3}{2}J_2\sqrt{\frac{\mu}{(a_{ij}^f)^7}}R_E^2 \cos i_{ij}^f, \tag{5}$$

where a_{ij}^f, i_{ij}^f, Ω_{ij}^f are, respectively, the averaged values of the semi-major axis, inclination, and LAN of the flight orbit of the *i*-th OOS spacecraft from the parking orbit (a^p, i^p, Ω_i^p) to the *j*-th destination orbit (a_j^d, i_j^d, Ω_j^d), R_E — the equatorial radius of the Earth, μ — the gravitational parameter of the Earth, J_2 — the coefficient at the second zonal harmonic of the Earth's geopotential.

Equations (3) and (4) have analytical solutions (6) and (7) [6, 8, 15]:

$$a_{ij}^f(t) = a^p\left(1 - \varepsilon_i \cos\tilde{\beta}_{ij}\sqrt{\frac{a^p}{\mu}}t\right)^{-2}, \tag{6}$$

$$i_{ij}^f(t) = i^p - \frac{2}{\pi}\tan\tilde{\beta}_{ij}\log\left(1 - \varepsilon_i \cos\tilde{\beta}_{ij}\sqrt{\frac{a^p}{\mu}}t\right). \tag{7}$$

The solution of equation (5) is determined by numerical integration using (6) and (7) and has the form (8):

$$\Omega_{ij}^f(t) = \Omega_i^p - \frac{3}{2}J_2\sqrt{\mu}R_E^2\int_0^t (a_{ij}^f(t))^{-7/2}\cos i_{ij}^f(t)\,dt. \tag{8}$$

Using (6), (7), we write down the boundary value problem for determining the controlling yaw angle $\tilde{\beta}_{ij}$ and the time of interorbital flight t_{ij}^{f} in the form of a system of transcendental equations (9), (10):

$$a_j^d = a^p\left(1-\varepsilon_i\cos\tilde{\beta}_{ij}\sqrt{\frac{a^p}{\mu}}\,t_{ij}^f\right)^{-2}, \qquad (9)$$

$$i_j^d = i^p - \frac{2}{\pi}\tan\tilde{\beta}_{ij}\log\left(1-\varepsilon_i\cos\tilde{\beta}_{ij}\sqrt{\frac{a^p}{\mu}}\,t_{ij}^f\right). \qquad (10)$$

The system of transcendental equations (9), (10) has an analytical solution (11):

$$\tilde{\beta}_{ij} = \begin{cases} \beta^{*}_{ij} & \text{if } a^p < a_j^d, \\ \pi+\beta^{*}_{ij} & \text{if } a^p > a_j^d \text{ and } i^p < i_j^d, \\ -\pi+\beta^{*}_{ij} & \text{if } a^p > a_j^d \text{ and } i^p > i_j^d, \end{cases} \qquad (11)$$

where

$$\beta^{*}_{ij} = \arctan\left[\frac{\pi(i^p - i_j^d)}{2}\left[\log\left(\sqrt{\frac{a^p}{a_j^d}}\right)\right]^{-1}\right].$$

The flight time t_{ij}^f of the *i*-th OOS spacecraft from the parking orbit to the *j*-th destination orbit is determined by the formula (12):

$$t_{ij}^f = \left(1-\sqrt{\frac{a^p}{a_j^d}}\right)\Bigg/\left(\varepsilon_i\cos\tilde{\beta}_{ij}\sqrt{\frac{a^p}{\mu}}\right), \qquad (12)$$

and the mass of fuel Δm_{ij} required for this flight is given by the expression (13):

$$\Delta m_{ij} = \frac{T_i}{c_i}t_{ij}^f, \qquad (13)$$

where c_i and T_i, accordingly, the fuel flow rate and engine thrust of the OOS spacecraft.

4. DETERMINATION OF WAITING TIME FOR THE START OF FLIGHTS

The use of the waiting of the OOS spacecraft on the parking orbits allows for eliminating the initial (at the time of service start) differences in the LAN of the parking orbit and the destination orbit due to the different angular velocities of nodal precession of these orbits. The duration of the wait depends significantly on the initial difference in the LAN of the parking orbits and the destination orbits of the OOS spacecraft. If the initial differences in the LAN of the parking and destination orbits of the OOS spacecraft are greater than the change in the LAN during the flight of the OOS spacecraft between these orbits, then the OOS spacecraft are waiting for the start of the flight. At the moments when the differences in the LAN of the parking and destination orbits of the OOS spacecraft are equal to the change in the difference in LAN during the flight, the flight between the parking and destination orbits of the OOS spacecraft begins.

Let us consider the angle s $\Delta\Omega_{ij}^0$, $\Delta\Omega_{ij}^f$, and $\Delta\Omega_{ij}^w$. The angle $\Delta\Omega_{ij}^0$ is equal to the initial difference of the LAN of the parking orbit of the *i*-th OOS spacecraft and the *j*-th destination orbit. The angles $\Delta\Omega_{ij}^f$ and $\Delta\Omega_{ij}^w$, respectively, are equal to the changes in the differences in the LAN of the parking orbit of the *i*-th OOS spacecraft and the *j*-th destination orbit during the flight time t_{ij}^f from the parking orbit to the destination orbit and during the waiting time t_{ij}^w of the OOS spacecraft in the parking orbit. The angle $\Delta\Omega_{ij}^0$ is measured in the direction of nodal precession of the parking and the destination orbits from the orbit with a higher angular velocity of nodal precession to an orbit with a lower angular velocity of nodal precession. Its value depends on the ratio of the values of Ω_i^p, Ω_j^d, and ω^p, ω_j^d, where Ω_i^p, Ω_j^d, and ω^p, ω_j^d are, respectively, the LAN and the angular velocity of nodal precession of the parking orbit of the *i*-th OOS spacecraft and the *j*-th destination orbit. Fig. 1 explains the calculation $\Delta\Omega_{ij}^0$ for the case $\Omega_j^d \geq \Omega_i^p$ and $\omega_j^d > \omega^p$.

Formulas for calculation $\Delta\Omega_{ij}^0$ in four possible cases are given in (14):

$$\Delta\Omega_{ij}^0 = \begin{cases} (\Omega_j^d - \Omega_i^p), & \text{if } \Omega_j^d \geq \Omega_i^p \text{ and } \omega_j^d > \omega^p, \\ 2\pi - (\Omega_i^p - \Omega_j^d), & \text{if } \Omega_j^d < \Omega_i^p \text{ and } \omega_j^d > \omega^p, \\ (\Omega_i^p - \Omega_j^d), & \text{if } \Omega_i^p \geq \Omega_j^d \text{ and } \omega^p > \omega_j^d, \\ 2\pi - (\Omega_j^d - \Omega_i^p), & \text{if } \Omega_i^p < \Omega_j^d \text{ and } \omega^p > \omega_j^d. \end{cases} \qquad (14)$$

As a result of the waiting flight from the parking orbit of the OOS spacecraft to the destination orbit, the difference in the LAN of these orbits should change due to $\Delta\Omega_{ij}^w$ and $\Delta\Omega_{ij}^f$ from $\Delta\Omega_{ij}^0$ to the allowable value for short-range guidance Ω_a.

According to the problem statement, the angular velocities of nodal precession of the parking orbits ω^p and angular velocities of nodal precession of the destination orbits ω_j^d of the OOS spacecraft are constant in magnitude and according to (5) are calculated using formulas (15) and (16):

$$\omega^p = -\frac{3}{2} J_2 \sqrt{\frac{\mu}{(a^p)^7}}\, R_E^2 \cos i^p \,, \tag{15}$$

$$\omega_j^d = -\frac{3}{2} J_2 \sqrt{\frac{\mu}{(a_j^d)^7}}\, R_E^2 \cos i_j^d \,. \tag{16}$$

The angle $\Delta\Omega_{ij}^f$ is calculated by the formula (17):

$$\Delta\Omega_{ij}^f = \left| -\frac{3}{2} J_2 \sqrt{\mu} R_E^2 \int_0^{t_{ij}^f} a_{ij}^f(t)^{-7/2} \cos\ i_{ij}^f(t)\, dt - \omega_j^d t_{ij}^f \right|, \tag{17}$$

where a_{ij}^f, i_{ij}^f, t_{ij}^f are calculated according to formulas (6), (7), (12).

The values

$$-\frac{3}{2} J_2 \sqrt{\mu} R_E^2 \int_0^{t_{ij}^f} a_{ij}^f(t)^{-7/2} \cos\ i_{ij}^f(t)\, dt$$

and $\omega_j^d t_{ij}^f$ are changes in the LAN of the flight orbit and the destination orbit during the flight of the OOS spacecraft from the parking orbit to the destination orbit accordingly.

The angle $\Delta\Omega_{ij}^w$ is determined by the following formula

$$\Delta\Omega_{ij}^w = \begin{cases} 0, & \text{if} \quad \left|\Delta\Omega_{ij}^0 - \Delta\Omega_{ij}^f\right| \le \Omega_a, \\ 2\pi - (\Delta\Omega_{ij}^f - \Delta\Omega_{ij}^0), & \text{if} \quad (\left|\Delta\Omega_{ij}^0 - \Delta\Omega_{ij}^f\right| > \Omega_a) \wedge (\Delta\Omega_{ij}^0 < \Delta\Omega_{ij}^f), \\ (\Delta\Omega_{ij}^0 - \Delta\Omega_{ij}^f), & \text{if} \quad (\left|\Delta\Omega_{ij}^0 - \Delta\Omega_{ij}^f\right| > \Omega_a) \wedge (\Delta\Omega_{ij}^0 > \Delta\Omega_{ij}^f). \end{cases}$$

The waiting time of the OOS spacecraft for the flight t_{ij}^w is determined by the formula (18):

$$t_{ij}^w = \Delta\Omega_{ij}^w \big/ \Delta\omega_j \,, \tag{18}$$

where $\Delta\omega$ is calculated by formulas (19) or (20):

$$\Delta\omega_j = \omega^p - \omega_j^d, \text{ if } \omega^p > \omega_j^d \,, \tag{19}$$

$$\Delta\omega_j = \omega_j^d - \omega^p, \text{ if } \omega^p < \omega_j^d \,. \tag{20}$$

The duration of the flight with the waiting of the *i*-th OOS spacecraft from the parking orbit to the *j*-th destination orbit is determined by the formula (21):

$$t_{ij} = t_{ij}^w + t_{ij}^f \,. \tag{21}$$

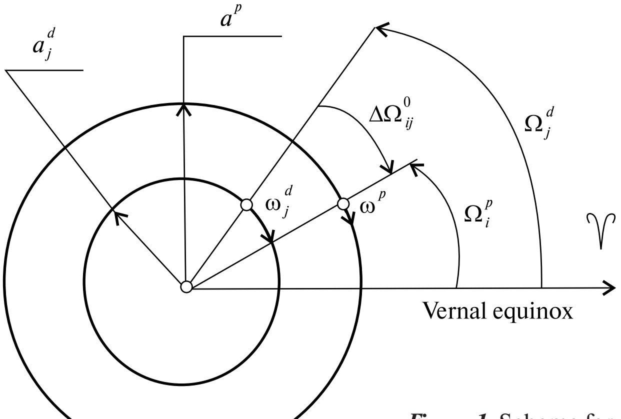

Figure 1. Scheme for calculation $\Delta\Omega_{ij}^0$

5. DISTRIBUTION OF REQUESTS FOR THE EXECUTION OF OOS

According to the problem statement, there are *m* requests for the execution of OOS. Requests can be performed by *n* spacecrafts of the OOS complex. *m* and *n* are natural numbers. To fulfill requests for OOS, each spacecraft of the OOS complex must fly from the parking orbit to the destination orbit. The waiting time for the *i*-th spacecraft of the OOS complex to start the flight from the parking orbit to the destination orbit to serve the *j*-th OOS client is estimated by the matrix t_{ij}^w . Matrix elements

$$t_{ij}^w \ \left(\forall i \in \{1, 2, .., n\}, \forall j \in \{1, 2, .., \cdots m\}\right)$$

are determined by the formula (18). Let's consider integer variables x_{ij} that take the value 0 or 1. Assume that $x_{ij} = 1$ if the *i*-th spacecraft of the OOS complex is assigned to fulfill the *j*-th request for execution of OOS and $x_{ij} = 0$ if the *i*-th spacecraft of the OOS complex is not assigned to run the *j*-th request for execution of OOS. In the problem, it is necessary to distribute the requests for execution of OOS among the spacecraft of the OOS complex in such a way that the total waiting time for the start of flights for the execution of all requests for execution of OOS is minimal. Taking into account the positive waiting times for the start of flights by spacecraft of the OOS complex, the minimum total waiting time for the start of flights by spacecraft of the OOS complex is equivalent to the minimum waiting times for each of the spacecraft of the OOS complex.

The problem of distribution of requests for the execution of OOS between spacecrafts of the OOS complex allows a mathematical formulation in the form of an integer linear programming problem with linear constraints (22):

$$x_{ij}^{*} = \underset{x_{ij}\in\{0,1\}}{\arg\min} \sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij}^{w}\, x_{ij}, \qquad (22)$$

where x_{ij}^{*} is the matrix of the optimal distribution of requests for performing OOS between the spacecrafts of the OOS complex. Depending on the ratio of values *m* and *n*, in addition to the condition $x_{ij} \in \{0,\ 1\}$, x_{ij} must satisfy various systems of linear equalities and inequalities. If $m < n$, it is assumed that all requests for implementing the OOS must be fulfilled, and some spacecrafts of the OOS complex may not perform the OOS. The mathematical formulation of these conditions has the form (23):

$$\begin{aligned} &\sum_{j=1}^{m} x_{ij} = 1,\ \forall i \in \{1, 2, ..., n\},\\ &\sum_{i=1}^{n} x_{ij} \le 1,\ \forall j \in \{1, 2, ..., m\}. \end{aligned} \qquad (23)$$

If $m > n$, then all spacecrafts of the OOS complex perform OOS, and some OOS requests may not be fulfilled. The mathematical formulation of these conditions has the form (24):

$$\begin{aligned} &\sum_{j=1}^{m} x_{ij} \le 1,\ \forall i \in \{1, 2, ..., n\},\\ &\sum_{i=1}^{n} x_{ij} = 1,\ \forall j \in \{1, 2, ..., m\}. \end{aligned} \qquad (24)$$

If $m = n$, then all spacecrafts of the OOS complex perform OOS and all requests for OOS execution must be fulfilled. The mathematical formulation of these conditions has the form (25):

$$\begin{aligned} &\sum_{j=1}^{m} x_{ij} = 1,\ \forall i \in \{1, 2, ..., n\},\\ &\sum_{i=1}^{n} x_{ij} = 1,\ \forall j \in \{1, 2, ..., m\}. \end{aligned} \qquad (25)$$

To solve the considered problem of integer linear programming with linear constraints, the method of branches and bounds was used.

6. SYNTHESIS OF THE ORBITAL STRUCTURE OF THE OOS COMPLEX

The synthesis of the orbital structure of the OOS complex includes the determination of the rational number of the OOS complex spacecrafts and the parameters of their parking orbits. According to the conditions of the problem, for the OOS complex spacecrafts' parking orbits, $\Omega_i^p\ \left(\forall i \in \{1, 2, .., n\}\right)$ are known. For the OOS complex spacecrafts' destignation orbits, there are known a_j^d, i_j^d, Ω_j^d $\left(\forall j \in \{1, 2, .., m\}\right)$. It is necessary to find the optimal values of the semi-major axis a_{opt}^p and the inclination i_{opt}^p of the OOS complex spacecrafts' parking orbits. The parameters a^p and i^p must satisfy the following constraints: $a_{\min}^p \le a^p \le a_{\max}^p$ and $i_{\min}^p \le i^p \le i_{\max}^p$. In constraints $a_{\max}^p$, $i_{\max}^p$ and $a_{\min}^p$, $i_{\min}^p$, respectively, are the maximum and minimum allowable values of the semi-major axis and the inclination of the OOS complex spacecrafts' parking orbits.The average duration of the maneuver waiting for the start of the flights t_{mid}^w and the average fuel consumption Δm_{mid} for the flights were taken as optimality criteria.

The problem of determining the optimal parameters (a_{opt}^p, i_{opt}^p) of the OOS complex spacecrafts parking orbits allows a mathematical formulation in the form of a bi-objective optimization problem (26):

$$(a_{opt}^p,\ i_{opt}^p) = \underset{(a^p,\, i^p)\in\vartheta}{\arg\min}\left(t_{mid}^w(a^p,\ i^p),\quad \Delta m_{mid}(a^p,\ i^p)\right), \qquad (26)$$

where the set of allowed values ϑ is formulated by a system of constraints (27):

$$\begin{aligned} &a_{\min}^p \le a^p \le a_{\max}^p,\\ &i_{\min}^p \le i^p \le i_{\max}^p. \end{aligned} \qquad (27)$$

This problem consists in finding a vector of target variables (a_{opt}^p, i_{opt}^p) satisfying the constraint (27) and optimizing the vector objective function

$$(t_{mid}^w(a^p, i^p),\quad \Delta m_{mid}(a^p, i^p)),$$

the elements of which

$$t_{mid}^w(a^p, i^p) \text{ and } \Delta m_{mid}(a^p, i^p)$$

are calculated according to formulas (28) and (29):

$$t_{mid}^w(a^p, i^p) = \sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij}^w x_{ij}^{*} / \min(n, m), \qquad (28)$$

$$\Delta m_{mid}(a^p, i^p) = \sum_{i=1}^{n}\sum_{j=1}^{m} \Delta m_{ij} x_{ij}^{*} / \min(n, m). \qquad (29)$$

The objective functions

$$t^{w}_{mid}(a^p, i^p) \text{ and } \Delta m_{mid}(a^p, i^p)$$

mutually conflict with each other because as the difference between a^p and a^d_j of the OOS complex spacecrafts increase, then t^{w}_{mid} decreases and Δm_{mid} increases. For the bi-objective optimization problem (26), there is no single solution that would simultaneously optimize both objective functions. Therefore, the Pareto principle was used for optimization. The Pareto principle [9, 10] does not single out a single solution; it only narrows the set of possible alternatives to a representative set of Pareto-optimal solutions (Pareto fronts). For solutions belonging to the Pareto front, none of the objective functions can be improved in value without worsening the other. The Pareto-optimal solution (a^p_{opt}, i^p_{opt}) must be chosen by the decision maker only from the Pareto front *P*, $(a^p_{opt}, i^p_{opt}) \in P$.

In the article, the computer approximation of the Pareto front is carried out using the genetic algorithm of global optimization. At each optimization step, the distribution problem was solved, and the matrix x^*_{ij} of the optimal distribution of requests for the execution of OOS between spacecrafts of the OOS complex was determined. The advantage of the genetic algorithm is that it generates a set of solutions that allow you to calculate an approximation of the whole Pareto front. In addition, it does not depend on the type of objective functions, the definition area, and the types of optimization variables do not require the setting of initial approximation and have a large number of control parameters.

The results of Pareto optimization are visualized by graphs of Pareto fronts (compromise curves). The compromise curves provide complete information on how the improvement of one objective function is related to the deterioration of another when moving along the compromise curve. The decision maker can take this information into account when determining the Pareto-optimal objective point.

The analysis of the performed calculations showed that an increasing number of spacecrafts of the OOS complex usually leads to a decrease in the average waiting time for the start of the interorbital flights by spacecrafts of the OOS complex. To determine the required number of OOS complex spacecrafts, the problem of determining the optimal parameters of the OOS complex spacecrafts' parking orbits is solved for a different number of spacecrafts in the OOS complex. Using the obtained graphs of the Pareto fronts, the decision maker determines the required number of the OOS complex spacecrafts.

7. CALCULATION EXAMPLE

Let us consider an example of the Pareto-optimal synthesis of the orbital structure of the OOS complex under acceptable restrictions on the average fuel costs and the minimum average waiting time for the start of flights. It is known that the average fuel consumption should not exceed 26 kg. The orbits of OOS clients have a small eccentricity and the following orbital parameters:

$$a^d_1 = 6978 \text{ km}, \; a^d_2 = 6878 \text{ km},$$

$$i^d_1 = 60.7°, \; i^d_2 = 59.6°,$$

$$\Omega^d_1 = 330°, \Omega^d_2 = 350°.$$

It is known that the parking orbits of the OOS complex spacecrafts have a small eccentricity, and their LAN are evenly distributed between 0° and 360° ($\Omega^p_1 = 0°$). The semi-major axis and inclination of the parking orbits a^p and i^p must satisfy the constraints $7178 \text{ km} \le a^p \le 7578 \text{ km}$ and $59° \le i^p \le 61°$. Two objective functions are used to optimize the param-

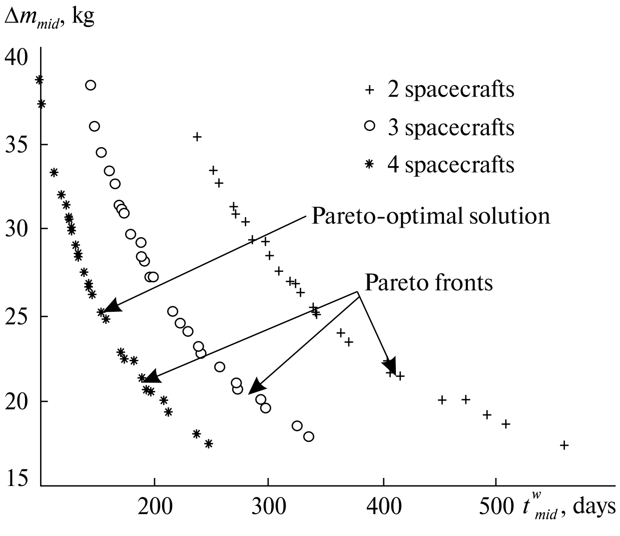

***Figure* 2**. Pareto fronts for different numbers of spacecraft in the OOS complex eters of the OOS complex spacecrafts parking orbits: the average duration of waiting for the start of flights $t^w_{mid}(a^p, i^p)$ and the average fuel consumption $\Delta m_{mid}(a^p, i^p)$ for flights of the OOS complex spacecrafts from parking orbits to the destination orbits. According to *the problem statement*, it is known that the OOS complex spacecrafts have propulsion systems with four stationary plasma engines SPT-140 with a total thrust $T = 1.2\ \text{N}$ and fuel flow rate of $20000\ \text{m/s}$. OOS complex spacecrafts have an initial mass of $1500\ \text{kg}$ and a payload mass of $500\ \text{kg}$.

Calculations of the Pareto fronts computer approximations were carried out for cases 2, 3, and 4 spacecrafts in the OOS complex and are shown in the graphs in Fig. 2.

As a Pareto-optimal solution to the problem, the point of the Pareto front with the following values of the objective functions was chosen:

$$t^w_{mid}(a^p_{opt}, i^p_{opt}) = 153.63\ \text{days}$$

$$\text{and } \Delta m_{mid}(a^p_{opt}, i^p_{opt}) = 25.15\ \text{kg}.$$

It corresponds to the minimum waiting time for the start of inter-orbital flights with acceptable fuel consumption. The selected values of the objective functions correspond to the following values of the $a^p_{opt} = 7335.7\ \text{km}$ and the inclination $i^p_{opt} = 60.58°$ of the OOS complex spacecrafts' parking orbits.

Using the parameters of the parking orbits of the OOS complex spacecraft, we determined: the matrix x_{ij} of the distribution of requests for the execution of OOS between the OOS complex spacecrafts, the matrix t^w_{ij} of the waiting time of the OOS complex spacecrafts for the start of flights, the matrix t^f_{ij} of the duration of flights of the OOS complex spacecrafts from the parking orbits to the destination orbits, the matrix fuel masses Δm_{ij}, and the matrix of control yaw angles β_{ij}, which are necessary for the flight of the OOS complex spacecrafts from the parking orbits to the destination orbits.

8. CONCLUSIONS

In the paper, the orbital structure of the low-orbit complex OOS in near-Earth orbits with a small eccentricity is proposed. The peculiarity of the proposed orbital structure is the placement of the OOS complex spacecrafts on the orbits with a small eccentricity and the same nodal precession speed. The planes of the parking orbits of the spacecraft of the OOS complex are evenly distributed by the LAN. The nodal precession speeds of the OOS spacecrafts' orbits and the orbits of the OOS clients differ significantly due to the difference in the major semiaxes of their orbits. For spacecrafts with engines of low constant thrust, the technique for optimal synthesis of the orbital structure and optimal operational planning of the low-orbit OOS space complex in orbits with low eccentricity has been developed. Several ballistic and optimization problems that arose during the development of the technique were formulated and solved. Methods for solving the problems are the averaging method, the branch-and-bound method, and the multi-objective optimization method. The use of the developed technique is illustrated by a calculation example. The technique can be used in the justification, planning, and design of OOS complexes in low Earth orbits with a small eccentricity.

REFERENCES

1. Alpatov A. P., Khoroshylov S. V., Maslova A. I. (2019). *Contactless de-orbiting of space debris by the ion beam. Dynamics and control.* Kyiv: Akademperiodyka, 170 p.
2. Alpatov A. P., Goldshtein Yu. M. (2021). Assessment perspectives for the orbital utilization of the space debris. *Space Science and Technology.* **27**, № 3. P. 3—12.
3. Barea A., Urrutxua H., Cadarso L. (2020). Large-scale object selection and trajectory planning for multi-target space debris removal missions. *Acta Astronaut.* **170***,* 289—301.
4. Chen H., Ho K. (2018). Integrated space logistics mission planning and spacecraft design with Mixed-Integer Nonlinear Programming, *J. Spacecraft and Rockets,* **55**, № 2, 365—381.
5. Dutta A., Arora N., Russell R. (2012). Peer-to-peer refueling strategy using low-thrust propulsion. *J. Spacecraft and Rockets,* **49**, 944—954.
6. Han C., Zhang S., Wang X. (2019). On-orbit servicing of geosynchronous satellites based on low-thrust transfers considering perturbations. *Acta Astronaut.* № 159, 658—675.
7. Ho K., Wang H., DeTrempe P. A., Jonchay T. S., Toniita K. (2020). Semi-analytical model for design and analysis of on-orbit servicing architecture. *J. Spacecraft and Rockets,* **57**, № 6, 1129—1138.

8. Holdshtein Yu. M. (2020). On the choice of a parking orbit for a service spacecraft. *Technical Mechanics,* № 3, 30—38 [in Ukrainian].
9. Kim I. Y., de Weck O. L. (2005). Adaptive weighted-sum method for bi-objective optimization: Pareto front generation. *Structural and multidisciplinary optimization,* **29**,149—158.
10. Popovici N. (2005). Pareto reducible multicriteria optimization problems. *J. Math. Programming and Operations Res.,* **54**, № 3, 253—263.
11. Razoumny Yu. N., Razoumny V. Yu., Spencer D. B., et al. (2017) The concept of on-orbit-servicing for next generation space system development and its key technologies. *Proceedings of the 68th International Astronautical Congress IAC,* **16**, 10486—10499.
12. Rousso P., Samsam S., Chhabra R. (2021). Amission architecture for on-orbit servicing industrialization. *IEEE Aerospace Conf.* (06—13 March, 2021), 1—14.
13. Shan M., Guo J., Gill E. (2016). Review and comparison of active space debris capturing and removal methods. *Progress in Aerospace Sci.,* **80**, 18—32.
14. Zhang Q., Sun F., Xu B., et al. (2012). Multiple spacecrafts on-orbit service task allocation based on DPSO. *Chin. Sp. Sci. Techno.,* **32**, 68—76.
15. Zhang S., Han C., Sun X. (2018). New solution for rendezvous between geosynchronous satellites using low thrust. *J. Guid. Contr. Dynam.,* **41**, № 3, 1397—1406.
16. Zhao Z., Hang J., Li H., Zhou J. (2017). LEO cooperative multi-spacecraft refueling mission optimization considering J 2 perturbation and target's surplus propellant constraint. *Adv. Space Res.,* **59**, 252—262.

Стаття надійшла до редакції 07.02.2023 *Received 07.02.2023*
Після доопрацювання 20.04.2023 *Revised 20.04.2023*
Прийнято до друку 21.04.2023 *Accepted 21.04.2023*

*Ю. М. Гольдштейн***,** старш. наук. співроб.**,** канд. техн. наук
ORCID.org/0000-0002-3931-2680,
E -mail: jura_gold@meta.ua

Інститут технічної механіки Національної академії наук України і Державного космічного агентства України
вул. Лешко-Попеля 15, Дніпро, Україна, 49005

МЕТОДИКА ОПТИМІЗАЦІЇ ОРБІТАЛЬНОЇ СТРУКТУРИ НИЗЬКООРБІТАЛЬНОГО КОМПЛЕКСУ ОРБІТАЛЬНОГО СЕРВІСНОГО ОБСЛУГОВУВАННЯ

Для виконання більшості місій орбітального сервісного обслуговування (ОСО), які плануються у цей час, використовуються одноразові космічні апарати ОСО. Використання одноразових космічних апаратів ОСО може бути вигідним в найближчому майбутньому. Але вона не є надійним розв'язком для ОСО в довгостроковій перспективі. В якості альтернативи більш корисною концепцією є використання багаторазових комплексів ОСО, які дозволяють реагувати на планові та випадкові запити клієнтів ОСО. Індустріалізація ОСО є логічним кроком розвитку космонавтики. Ця концепція може забезпечити своєчасність і оперативність здійснення ОСО при планових обслуговуваннях і випадкових запитах клієнтів ОСО. Однак, незважаючи на потенційну перевагу багаторазового ОСО, проєктування його орбітальної структури і операційного обслуговування значно складніше і важче в порівнянні із традиційною концепцією організації ОСО. Це пов'язано з тим, що при плануванні реагування багаторазових комплексів ОСО на запити необхідно проводити розподіл операцій обслуговування клієнтів ОСО між космічними апаратами багаторазового комплексу ОСО. Зараз космічна галузь перемикає свою увага на область низьких навколоземних орбіт. Це зумовлює збільшення розгорнутих і запланованих до розгортання низькоорбітальних супутникових угрупувань, кількості супутників у них, відмінністю структурних схем супутникових угрупувань і суттєвим впливом навколишнього середовища на орбітальні параметри. Як відомо, орбітальні параметри низьких орбіт космічних апаратів можуть суттєво відрізнятися і різниця між ними може досягати за довготою висхідного вузла десятків і навіть сотень градусів. Це призводить до неприпустимо великих для сучасних космічних апаратів ОСО енергетичних витрат для активного повороту площин їх вихідних орбіт до площин орбіт призначення. У низці робіт розглянуто можливість зменшення цих енергетичних витрат за рахунок використання різниці швидкостей прецесії лінії вузлів орбіт базування і призначення космічного апарата ОСО внаслідок нецентральності гравітаційного поля Землі. Однак за рахунок тривалого очікування космічного апарата ОСО на орбіті базування суттєво зростає час виконання перельоту з очікуваннямміж орбітами базування і призначення. Його скорочення можливо досягти за рахунок збільшення кількості та раціонального вибору висоти і нахилу орбіт базування космічних апаратів ОСО. Метою статті є розробка методики для оптимального синтезу орбітальної структури та оптимального операційного планування низькоорбітального комплексу ОСО на навколоземних орбітах з малим ексцентриситетом. Методами розв'язку задачі є метод усереднення, метод гілок і меж та метод багатокритеріальної оптимізації. Новизна отриманих результатів полягає розробці методики для оптимального синтезу орбітальної структури та оптимального операційного планування низькоорбітального космічного комплексу ОСО на навколоземних орбітах з малим ексцентриситетом. Розроблена методика може бути використана при обгрунтуванні, плануванні і проєктуванні космічних комплексів ОСО на низьких навколоземних орбітах з малим ексцентриситетом.

Ключові слова: багатоцільова оптимізація, орбіта базування, границя Парето, орбітальне сервісне обслуговування, мала тяга, метод усереднення.