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RELATIVE MOTION CONTROL SYSTEM OF SPACECRAFT FOR CONTACTLESS SPACE DEBRIS REMOVAL

Introduction. *The research deals with the development of a spacecraft control system for contactless space debris removal using the "ion beam shepherd" technology. Such a system is necessary to provide conditions for effective transfer of decelerating impulse to a space debris object by ion beam in the deorbiting phase.*

Problem Statement. *The design and analysis of the system has to be carried out taking into account the ion beam effects, a wide range of orbital disturbances, inaccuracies in determining the relative position and implementing the control actions, time-varying and parametric uncertainty, and limitations on the control actions.*

Purpose. *The purpose is to design a system to control spacecraft relative motion for contactless space debris removal.*

Materials and Methods. *The mixed sensitivity approach is applied to the system design. The requirements for the controller are specified in the frequency domain using the selected weight functions. The structured singular values methodology is used to analyze the system robustness.*

Results. *The system robustness and compliance with specified requirements have been confirmed both by a formal criterion and by computer simulation. A rational softening of the requirements for the control accuracy enables reducing significantly the propellant mass needed to maintain the relative position keeping an acceptable rate of space debris removal.*

Conclusion. *The designed control system provides a compromise between robust stability, performance, and costs of control under the impact of a wide range of disturbances.*

Keywords: control system, relative motion, space debris, ion beam shepherd, and robust stability.

Today, the near-Earth pollution with space debris (useless artificially created objects in space, such as spent rocket stages, defunct spacecraft, etc.) has reached a critical level [1]. In this regard, the space community is actively studying ways of direct removal of orbiting bodies from near-Earth orbits to solve the problem of space debris. The concept of contactless deorbiting known as «Ion Beam Shepherd» (IBS) [2] has several advantages (such as efficiency, low risk, reusability, and technological readiness) over other well-known approaches.

The IBS concept presupposes the installation of primary and secondary propulsion systems on a spacecraft (SC) that is called «ion-beam shepherd» (IBS SC). The ion beam from the ion thruster of primary propulsion system (PPS) is targeted to an orbiting body (OB) and is used to transmit to it a braking force impulse. The secondary propulsion system (SPS) i.e. another ion thruster is directed in such a way as to compensate the reaction force of the primary propulsion system (Fig. 1).

In order to ensure effective deorbiting, it is necessary to keep a certain constant distance between the shepherd spacecraft and the target body,

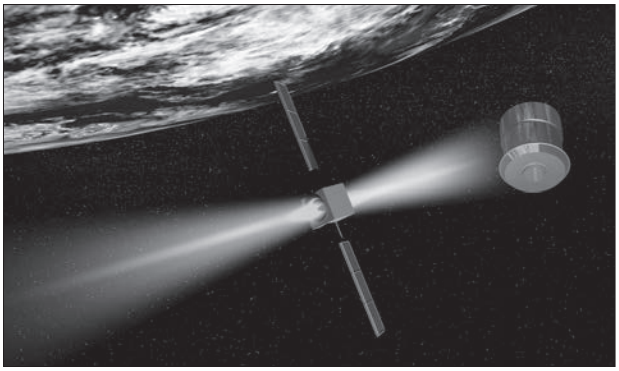

Fig. 1. IBS conception scheme

which should not exceed several diameters of deorbited body. It should be noted that only to compensate the reaction force of the PPS is not enough, insofar as uncontrolled relative motion of the formation is unstable. Therefore, the IBS SC must have a system for controlling its center-of-mass motion with respect to OB [2]. The development of this control system is complicated by many factors. The most important of them are as follows:

- space debris is bodies that do not cooperate with IBS SC. As a result, usually, OB weight and the vector of state cannot be measured with a high accuracy;
- in the general case, the mathematical model of relative dynamics is unstable;
- while designing the system it is important to reach a compromise between robust stability, performance, and propellant consumption for keeping the required relative position.

For the IBS technology being a rather new one, today, there have been a limited number of publications on various aspects of its development and only a few studies of the control system. Therefore, for designing the IBS SC control system it is advisable to use results of studying other similar problems. The most similar to the given problem is control of spacecraft for approaching the target body and further docking. For example, the research [3] has presented results of designing a control system for berthing to orbital station. The authors [4] have used μ-synthesis to design a system for approaching and docking to the objects that are not equipped with special means for these missions.

In addition to these studies, there have been several publications on control engineering, which can be useful for solving the abovementioned problem. The research [5] contains practical results for multivariable feedback control design in conditions of uncertainty. The authors [6] have made H_2-synthesis with parametrical uncertainty of plant taken into consideration. The problem of robust performance for the plant with time-variable uncertainties has been studied in [7]. In the research [8], an extended state observer has been used to control a plant.

Among the limited list of studies dealing directly with IBS it is necessary to point out the research [2], where the problem of IBS SC relative motion control has been formulated and its complexity and importance has been emphasized. In the research [9], an original method for determination of the force transmitted by the ion beam, which can be implemented directly on the board of IBS SC with the use of photo camera has been proposed [10]. In the research [11] the results of validation of this method have been given. The study [12] deals with dynamics and control of IBS SC. Its results are based on assumptions that the relative motion parameters are measured error-free, and OB weight is precisely known. However, the mentioned assumptions put essential limitations on the use of these results.

Below, the results of synthesis and analysis of a system for controlling IBS SC motion with respect to OB. The task has been solved with necessary compromise between robust stability, performance, and control costs and specific features of effects transmitted by the ion beam, external perturbations, inaccuracy of relative position determination, and flaws of propulsion system bodies.

The main role of the control system is to keep given position of IBS SC with respect to OB during its deorbiting from quasi-circular low near-Earth orbit.

To measure the coordinates of vector that determines the IBS SC position with respect to OB, the system contains sensors based on *LIDAR* (*Light Identification Detection and Ranging*) technology.

The actuators of control system are hydrazine thrusters (HT) with pulse width modulator (PWM) of the thrust. The pulse duration is calculated as follows:

$$t_{on} = \frac{F}{F_{th}} T, t_{on} \le T,$$

where F is output value of the controller; F_{th} is nominal thrust of HT; T is discretization period of the controller.

Despite the fact that PWM is a discrete device, it does not have any adverse effect on bandwidth and stability margins of the system, only adds additional damping [13]. In this regard, the PWM should be excluded from the system at the stage of synthesis of the controller limiting to consideration of the given accuracy of control actions implementation. At the same time, at the final stage, the synthesized controller will be validated using the nonlinear dynamics model of the IBS SC — OB system with PWM taken into account.

The system synthesis and analysis of dynamics are made for input data given in Table 1.

Table 1

Input Data

Parameters	Value
Initial orbit altitude, km	640
Final orbit altitude, km	340
Orbit inclination, degree	90
Eccentricity	0...0.05
IBS SC weight, kg	500 ± 50
OB weight, kg	1575 ± 315
Accuracy of determination of space debris position in orbital system of coordinates for each axis, m	≤0.5
Nominal thrust of PPS, N	0.031
Discretization period of control system, s	1
Nominal thrust of reactive actuators, N	2
Minimal impulse of reactive thrust of actuators, Ns	0.01

MATHEMATICAL MODEL OF RELATIVE DYNAMICS

For mathematical description of IBS SC — OB motion, an orbital *Oxyz* coordinate system is used. The origin of the orbital coordinate system (OCS) is placed in the center of mass of IBS SC. The *Ox* axis coincides with the direction of the radius vector that determines the center of mass of IBS SC with respect to the Earth center of mass. The *Oz* axis coincides with the normal to the plane passing through the axis and the vector of IBS SC orbital velocity and is directed towards the positive values of orbital kinetic moment. The *Oy* axis complements the system to the right coordinate system.

The OB position with respect to OCS is determined by radius vector L. The relative dynamics of IBS SC — OB system can be described using the following linearized system of equations [14]:

$$\begin{aligned} &\ddot{x} - \omega^2 x - 2\omega\dot{y} - \dot{\omega}y - kx = \frac{f_x^d}{m^d} - \frac{f_x^s}{m^s}, \\ &\ddot{y} - \omega^2 y + 2\omega\dot{x} + \dot{\omega}x + ky = \frac{f_y^d}{m^d} - \frac{f_y^s}{m^s}, \\ &\ddot{z} + kz = \frac{f_z^d}{m^d} - \frac{f_z^s}{m^s}, \end{aligned} \quad (1)$$

where x, y, z are L projections on OCS axes; , m^s, m^d are weights of IBS SC and OB, respectively; f_x^d, f_y^d, f_z^d are projections of resulting vector F^d of forces acting on OB on the OCS; f_x^s, f_y^s, f_z^s are projections of resulting vector F^s of forces acting on IBS SC on the OCS axes; ω, $\dot{\omega}$ and k from (1) are defined as:

$$\omega = \sqrt{\frac{\eta}{p^3}}(1 + \varepsilon\cos\upsilon), \; p = a(1 - \varepsilon^2),$$

$$\dot{\omega} = -2\varepsilon\sqrt{\frac{\eta}{p^3}}\sin\nu(1 + \varepsilon\cos\upsilon)\omega,$$

$$k = \frac{\eta}{R^3}, \; R = \frac{a(1 - \varepsilon^2)}{1 + \varepsilon\cos\upsilon},$$

where η is gravity constant of Earth; υ is true anomaly; ε is eccentricity; a is major semi-axis.

The forces F^d and F^s may include disturbances due to the noncentral part of the Earth's gravitational field, the attraction of the Sun and the Moon, atmospheric drag, solar pressure, in addition, the forces of PPS and SPS act on the IBS SC, and ion beam transmitted force acts on the OB.

The first and second equations of system (1) describe dynamics of the system in the orbit plane, while the third one determines its motion out of the orbit plane.

ION BEAM FORCE

Ion beam force is a specific disturbance typical for the studied system. Insofar as results of studies of these disturbances have not been sufficiently described in publications let us consider in detail the specific features of accounting the ion beam effect for solving the control-related tasks.

Based on several assumptions, the elementary force transmitted to OB can be calculated in the following way [9]:

$$dF_I^d = mnU(-V \cdot U)ds, \tag{2}$$

where m is weight of particle; U is particle velocity vector; ds is elementary area of OB surface; V is normal unit vector to the elementary area.

Force F_I^d transmitted by ion beam to OB can be calculated by integrating the elementary forces (2) over irradiated surface S,

$$F_I^d = \int_S dF_I^d .$$

Using the dimensionless similarity function $h(\tilde{z})$, the plasma density in arbitrary point with coordinates r, z can be determined in the following way [9]:

$$n = \frac{n_0}{h^2(\tilde{z})} \exp\left(-C \frac{\tilde{r}^2}{2h^2(\tilde{z})}\right), \ \tilde{r} = r / R_0 ,$$

where n_0 is plasma density at the beginning of far region of the beam; C is coefficient that determines which part of plasma flux falls into the circle of radius R_0 (for instance, $C = 3$ corresponds to 95 % beam flux hit).

At $M_0 \geq 40$ and a distance to OB less than 10 meters the plasma distribution can be considered conical. In this case, the similarity function can be determined as follows:

$$h = \tilde{z} \tan \alpha_0 ,$$

where α_0 is divergence angle of the cone.

For this problem it can be assumed that the axial component of plasma ion velocity is almost constant:

$$u_z = u_{z0} = \text{const},$$

where as the radial component of velocity is determined by the formula [10]:

$$u_r = u_z \tilde{r} \frac{h'}{h} .$$

In [10] it has been shown that to calculate the force one can consider the central projection of OB instead of 3D surface, which enables simplifying the understanding of effect of this factor on the dynamics of relative motion.

In the cases when the whole beam falls in OB, the force transmitted is approximately equal to:

$$F_I^d \approx [0 \ -f_T \ 0]^T, \tag{3}$$

where f_T is SPP thrust.

The action of this force decreases in y-direction and insignificantly increases in x- and (or) z-direction, when the beam cone partially passes through OB. Since the key task of controlling the relative position of IBS SC is to ensure as complete as possible beam hit on OB, one can conclude that disturbances caused by ion beam are limited and slightly differ from (3).

CONTROLLER SYNTHESIS

To synthesize the controller, it is advisable to use the H_∞ methodology [15]. This approach enables to synthesize a controller that minimizes the output of closed-loop system in the worst case of input disturbances. To use this approach, the object (1) is presented in the following standard form:

$$\begin{aligned} \dot{X} &= AX + B_1 w + B_2 u , \\ z &= C_1 X + D_{11} w + D_{12} u , \\ v &= C_2 X + D_{21} w + D_{22} u , \end{aligned} \tag{4}$$

where w is disturbance; u is control; z is output to be minimized; v is measured output.

The following disturbances are considered as input signals: w_1 is external disturbances, w_2 is vector L values, w_3 is error of vector L measurement, w_4 is error of control actions realization.

The vector of output to be minimized should contain the error of keeping the given position of IBS SC with respect to OB z_1 and the actuation error z_2.

Dependence between input w and output z is as follows:

$$z = F_l(P, K)w,$$

where P is transfer function of the controlled object (CO), K is controller's transfer function, and $F_l(P,K)$ is transfer function of the closed system.

The controller can be determined based on the condition of minimization of $\|H\|_\infty$ that is the norm of transfer function of the closed-loop system [15]:

$$\|F_l(P, K)\|_\infty \to \min.$$

In order to ensure the required performance and to limit the controller effort, the initial system should be added with weight functions $W_1(s)$ and $W_2(s)$ in such a way as it is shown in Fig. 2.

To weight the output z_1, low-frequency filters of the 1st order were used in the following form:

$$W_{1k}(s) = \frac{s/M_{1k} + \Omega_{1k}}{s + A_{1k}\Omega_{1k}},\ k = x, y, z. \quad (5)$$

The parameters Ω_{1k} are chosen based on the required controller bandwidth. The required steady-state error can be ensured by choosing the parameters A_{1k}, while the parameters M_{1k} enable to limit the overshoot.

The weight functions for control signal are similar to those in (5):

$$W_{2k}(s) = \frac{s/M_{2k} + \Omega_{2k}}{s + A_{2k}\Omega_{2k}},\ k = x, y, z. \quad (6)$$

The filter parameters (6) are chosen in such a way as to limit in a certain way the control at low frequencies and to minimize it at high ones. Such a choice of mentioned parameters enables limiting consumption of PPS propellant and reducing sensitivity to high-frequency noises.

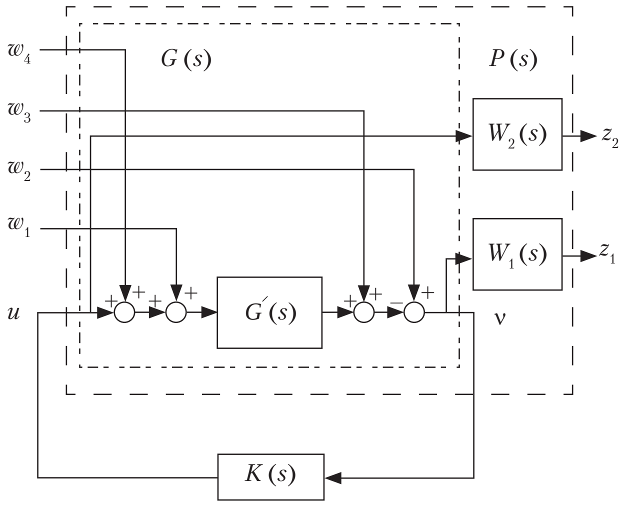

Fig. 2. Block diagram of augmented plant

Unlike the conventional method of mixed sensitivity, the augmented plant (Fig. 2) does not contain weight functions that approximate the CO uncertainty. This enables to reduce dimensionality of the controller. The specificity of proposed approach is that to ensure the required amplitude and frequency characteristics of functions of sensitivity S, input sensitivity KS, and complementary sensitivity T, i.e. required indices of system quality and robustness, only two weight matrixes $W_1(s)$ and $W_2(s)$ are used. The robust stability and robust quality are confirmed below, after the controller synthesis using an additional formal criterion.

In the case of control synthesis in the orbit plane the matrixes (3) are presented as follows:

$$X = [x, y, \dot{x}, \dot{y}]^T,\ w = [f_x, f_y, x_r, y_r, \Delta x, \Delta y, \Delta u_x, \Delta u_y]^T,\ u = [u_x, u_y]^T,$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \omega^2 + 2k & \dot{\omega} & 0 & 2\omega \\ -\dot{\omega} & \omega^2 - k & -2\omega & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{f}_x^{\max} & 0 & 0 & 0 & 0 & 0 & -F_{th}t_{on}^{\min}/Tm^s & 0 \\ 0 & \tilde{f}_y^{\max} & 0 & 0 & 0 & 0 & 0 & -F_{th}t_{on}^{\min}/Tm^s \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1/m^s & 0 \\ 0 & -1/m^s \end{bmatrix},\ C_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ C_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0 & 0 & 1 & 0 & -\Delta x^{\max} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\Delta y^{\max} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0 & 0 & 1 & 0 & -\Delta x^{\max} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\Delta y^{\max} & 0 & 0 \end{bmatrix},\ D_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where $\Delta x^{\max}$, $\Delta y^{\max}$ are maximum errors of relative position in the channels x and y, respectively.

Based on maximum difference in accelerations caused by the action of external disturbances and ion beam on the studied system, the following values of B_1 matrix elements were chosen:

$$\tilde{f}_x^{\max} = 3 \cdot 10^{-7}\ \text{N/kg},\ \tilde{f}_x^{\max} = 4.722 \cdot 10^{-5}\ \text{N/kg}.$$

Elements of matrixes D_{11} and D_{21} are determined based on the condition of maximum error of OB relative position:

$$\Delta x^{\max} = 0.5\ \text{m},\ \Delta x^{\max} = 0.5\ \text{m}.$$

The parameters $\Omega_{1x} = \Omega_{1y}$ 5 ω/π Hz of weight functions $W_{1k}(s)$ enable to ensure the required bandwidth of the controller, and the values $A_{1x} = A_{1y} = 0.1$ make it possible to get in the steady-state error not exceeding 10% of desired values of vector L. The parameters $M_{1x} = M_{1y} = 2$ limit the overshoot to 30%.

For the weight functions of control signal the following parameters are chosen:

$$M_{1x} = M_{1y} = 0.1;\ A_{1x} = A_{1y} = 10;$$
$$\Omega_{2x} = \Omega_{2x} = 20\,\Omega_{1x}.$$

They enable to put certain limitations on the control at low frequencies and to minimize it at high frequencies in order not to track high-frequency noises.

Relationship between the matrix transfer functions $G_{ij}(s)$ and the object representation $G(s)$ in the state space (3) is determined by the expression:

$$G_{ij}(s) = C_i(sI - A)^{-1} B_j + D_j,\ i, j = 1, 2.$$

The transfer function of the augmented plant is defined by the expression:

$$P(s) = G(s)\ W(s), \qquad (7)$$

Where $W(s)$ is diagonal matrix with the following diagonal elements:

$$W_{11}(s) = W_{1x}(s),\ W_{22}(s) = W_{1y}(s),$$
$$W_{33}(s) = W_{2x}(s),\ W_{44}(s) = W_{2y}(s).$$

To synthesize control over IBS motion in the direction perpendicular to the orbit plane, the matrixes of representation (4) are as follows:

$$X = [z, \dot{z}]^T,\ W = [f_z, z_r, \Delta z, \Delta u_z,]^T,\ U = [u_z]^T,$$

$$A = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \tilde{f}_z^{\max} & 0 & 0 & -F_{th}t_{on}^{\min}/Tm^s \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ -1/m^s \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix},\ C_2 = [-1 \quad 0],$$

$$D_{11} = \begin{bmatrix} 0 & 1 & \Delta z^{\max} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ D_{12} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ D_{21} = [0 \quad 1 \quad -\Delta z^{\max} \quad 0],\ D_{22} = [0],$$

where $\Delta z^{\max}$ is maximum errors of OB relative position measurements in the channel z.

Based on the same considerations, like in the case of motion in the orbit plane, the element $\tilde{f}_z^{\max}$ of matrix B_1 is equal to:

$$\tilde{f}_z^{\max} = 15 \cdot 10^{-7}\ \text{N/kg}.$$

The elements of matrixes D_{11} and D_{21} are determined based on the condition of maximum error of OB relative position measurement in the following way:

$$\Delta z^{\max} = 0.5\ \text{m}.$$

In this case, the matrix $W(s)$ has the following diagonal elements $W_{11}(s) = W_{1z}(s)$, $W_{22}(s) = W_{2z}(s)$.

For the object (5), using the known algorithms [16] based on solution of linear matrix inequalities, a sub-optimal controller $K(s)$ has been synthesized:

$$\dot{X}_K = A_K X_K + B_K v,$$

$$u = C_K X_K + D_K v,$$

provided

$$\|F_l\,(P, K)\|_\infty \le \gamma_{\min}.$$

To control the relative motion of IBS SC in the orbit plane, the matrixes A_K, B_K, C_K, D_K of sub-optimal controller of the 8th order for $\gamma_{\min} = 0.727$ have been found. For the motion out of the plane, a 4th order controller for $\gamma_{\min} = 0.695$ has been synthesized.

For the mentioned controllers, Figs. 3, 4 show sensitivity, complementary sensitivity, and input sensitivity functions. As one can see, the synthesized controllers meet the requirements stated using weight functions.

At the final stage, the synthesized controllers have been transformed into the discrete form using bilinear transformations. The norm of H_∞-controller is kept for this transformation method.

SYSTEM ROBUSTNESS ANALYSIS

As shown above, the controller is synthesized at nominal values of mathematical model parameters. At the same time, in real conditions, these parameters might differ from the nominal ones. For example, a precise weight of OB is unknown, IBS SC weight decreases as PPS propellant is consumed, and the mathematical model coefficients (1) vary as orbit altitude lowers. In addition, orbit eccentricity also can vary while deorbiting. In this case, the mathematical model coefficients depend exclusively on true anomaly. The-

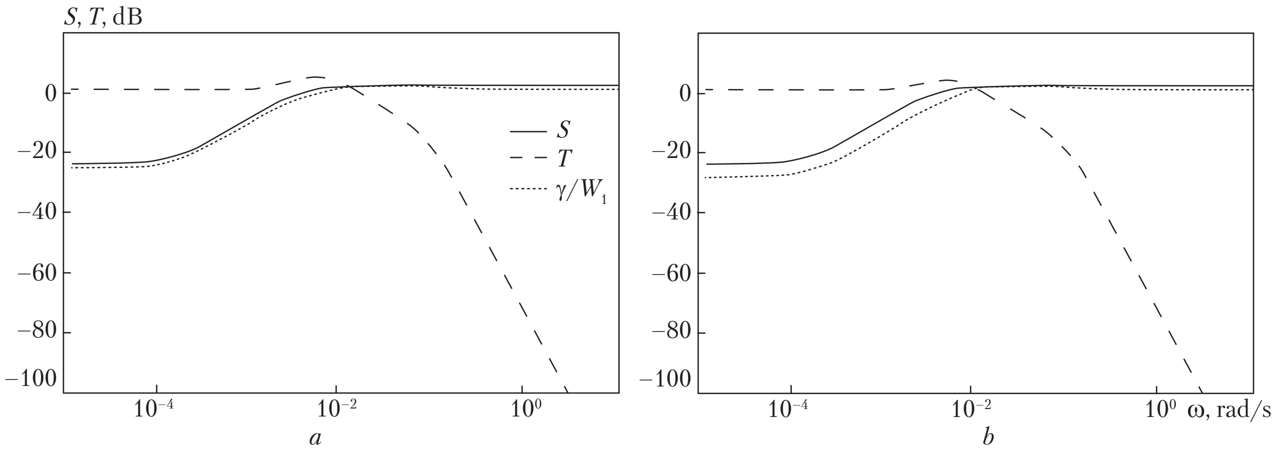

Fig. 3. Sensitivity and complementary sensitivity functions of CO with controller: *a* — motion in orbital plane; *b* — motion out of orbital plane

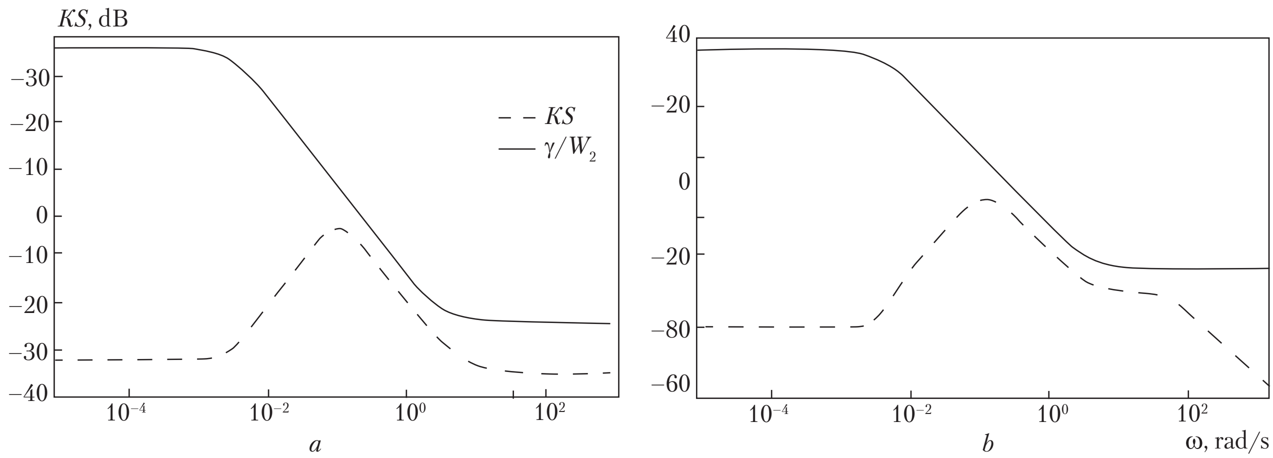

Fig. 4. Input sensitivity of CO with controller: *a* — motion in orbital plane; *b* — motion out of orbital plane

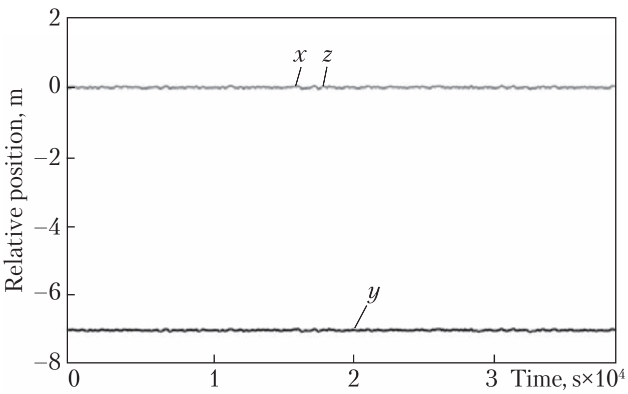

Fig. 5. Change in relative position of SC-IBS

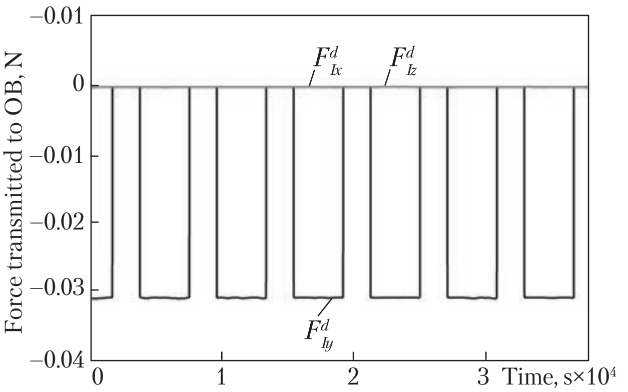

Fig. 6. Change in force transmitted by ion beam to OB

refore, it is necessary to analyze the effect of these factors on stability and performance.

Given the fact that while deorbiting OB the coefficients in the equations (1) change smoothly, the analysis of stability of object with variable coefficients can be replaced by the study of robust stability of the system with respect to uncertain parameters m^s, m^d, ω, $\dot{\omega}$, k.

Using the linear fractional transformation [15] the uncertain parameters of the model $\omega = \omega_n \pm d\omega$, $\dot{\omega} = \dot{\omega}_n \pm d\dot{\omega}$, $k = k_n \pm dk$, $m^s = m^s_n \pm dm^s$, $m^d = m^d_n \pm dm^d$ are presented as:

$$\omega = \omega_n + d\omega\Delta_1 = F_L(M_\omega, \Delta_1),$$

$$\dot{\omega} = \dot{\omega}_n + d\dot{\omega}\Delta_2 = F_L(M_{\dot{\omega}}, \Delta_2),$$

$$k = k_n + dk\Delta_3 = F_L(M_k, \Delta_3),$$

$$m^s = m^s_n \pm dm^s\Delta_4 = F_L(M^s_m, \Delta_4),$$

$$m^d = m^d_n \pm dm^d\Delta_5 = F_L(M^d_m, \Delta_5),$$

where $F_L(M, \Delta)$ is notation to indicate that the lower loop of matrix transfer function M is closed with matrix Δ;

$$M_\omega = \begin{bmatrix} \omega_n & d\omega \\ 1 & 0 \end{bmatrix};$$

$$M_{\dot{\omega}} = \begin{bmatrix} \dot{\omega}_n & d\dot{\omega} \\ 1 & 0 \end{bmatrix};$$

$$M_k = \begin{bmatrix} k_n & dk \\ 1 & 0 \end{bmatrix};$$

$$M^s_m = \begin{bmatrix} m^s_n & dm^s \\ 1 & 0 \end{bmatrix};$$

$$M^d_m = \begin{bmatrix} m^d_n & dm^d \\ 1 & 0 \end{bmatrix};$$

$$\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5 \in [-1, 1]$$

The parameters m^s and m^d are included into the mathematical model in the inverse form. In this case, the linear fractional transformation is as follows:

$$\left(m^{s(d)}\right)^{-1} = \left(F_L\left(M^{s(d)}_m, \Delta_{4(5)}\right)\right)^{-1} = F_L\left(\tilde{M}^{s(d)}_m, \Delta_{4(5)}\right),$$

where $\tilde{M}^{s(d)}_m = \begin{bmatrix} \left(m^{s(d)}_n\right)^{-1} & -dm^{s(d)}\left(m^{s(d)}_n\right)^{-1} \\ \left(m^{s(d)}_n\right)^{-1} & -dm^{s(d)}\left(m^{s(d)}_{n\ n}\right)^{-1} \end{bmatrix}$.

Using this representation of mathematical model parameters, the structural scheme of the system as shown in Fig. 2 can be presented as system consisting of block N (nominal plant and controller) and disturbance Δ structured and having the following block diagonal form:

$$\Delta = \begin{bmatrix} \Delta_1 & 0 & \cdots & 0 \\ 0 & \Delta_2 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \Delta_5 \end{bmatrix}.$$

In the case of structured uncertainty, it is advisable to apply the robustness measure that uses structural singular value [15]. The structured singular value for complex-valued matrix M is inverse

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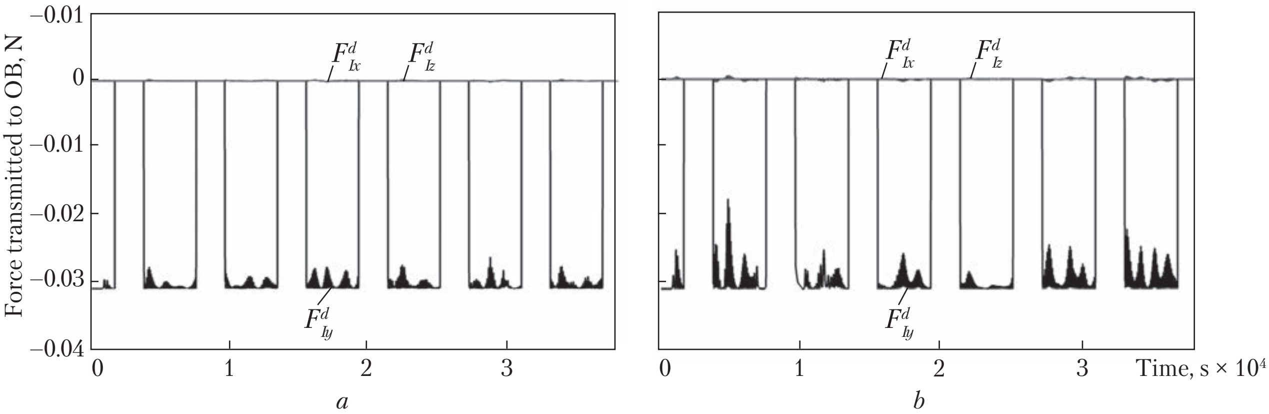

Fig. 7. Change in force transmitted by ion beam to OB in the case of softened requirements for accuracy of control: *a* — at a relative position error < 0.5 m; *b* — at a relative position error < 0.75 m

norm of the least disturbance from the considered class D, which makes the matrix $I + M\Delta$ singular. The structured singular value $\mu(M)$ is determined as follows:

$$\frac{1}{\mu(M)} = \inf_{\Delta \in D, \det(I+M\Delta)=0} \overline{\sigma}(\Delta) .$$

Let us assume that satisfactory performance is ensured by the condition $\left\| N^{\Delta} \right\|_{\infty} \leq 1$, where N^{Δ} is transfer function from w to z.

It is known [15] that the system has robust stability and robust performance with respect to all structured disturbances only if $\mu(N) < 1$.

Taking into consideration the parameters given in Table 1, for this system with controller, the maximum structured singular values are equal to 0.745 and 0.649 for the motion in and from the orbit plane, respectively. This enables to conclude that the synthesized controller ensures robust stability and robust performance, with the considered variations of parameters of the system mathematical model taken into account.

IBS SC RELATIVE MOTION SIMULATION

To verify the synthesized system of control by computer modelling, well-known nonlinear equations that describe motion of satellite in the central gravitational field considering the disturbances acting on it have been used. The disturbances include non-central part of the Earth gravitational field, Sun and Moon gravitation, atmospheric drag, solar pressure, PPS and SPS forces, and the force transmitted by ion beam to OB. IBS SC is assumed to be oriented in such a way as the PPS thrust vector is directed tangentially to the orbit. The simulation takes into consideration PPS and SPS shutdown while IBS SC moves in the shadow area.

Table 2

Total Impulse of PPS Thrust and OB Deorbiting Rate at Different Accuracy of IBS SC Control

No.	Maximum error of control, m	Total impulse of PPS thrust, Ns	OB deorbiting rate, km/day
1	0.1	27 302	1031
2	0.25	4 801	1031
3	0.5	2 021	1023
4	0.75	1 785	986

While calculating the force transmitted by ion beam, OB was simulated as a sphere having a radius of 1.32 m. The mass center is shifted by 35.5 cm along axis Qz with respect to the geometrical center.

The forces and torques transmitted by ion beam to OB were calculated using algorithms given in [17].

The OB motion around the mass center was simulated using known equations describing angular motion of solid absolutely rigid body under the action of disturbances [18]. The disturbances of angular motion include effects of gravitational and aerodynamic torques, as well as moments of solar pressure force and ion beam force.

Noises of measurements of relative position and SPS error were simulated as random values with Gaussian distribution.

Figs. 5–8 feature the results of simulation for OB orbit with an initial eccentricity $\varepsilon = 0.002$. Weights of IBS SC and OB were estimated as $m^s = 450$ kg and $m^d = 1890$ kg, respectively.

Fig. 5 shows that the control system enables to ensure the required position of IBS SC with respect to OB with an accuracy of up to 0.1 m. Also, the simulation results show that in the lightened areas of orbit, control effect is stably transferred from PPS to OB (Fig. 6). The calculations for various options of input data given in Table 1 show that the error of IBS SC relative motion control does not exceed 0.2 m.

As the simulation results show, the synthesized controller enables to ensure a precise IBS SC positioning with respect to OB, however for this a material consumption of PPS propellant is required. Softening of requirements for accuracy of control makes it possible to reduce propellant consumption, however, this may lead to slowing down the OB deorbiting rate as a result of the fact that a large portion of PPS ions do not fall on its surface (Fig. 7).

Table 2 contain values of total impulse of PPS thrust and OB deorbiting rate obtained from computer modelling of deorbiting within 3.8 days for various errors of control of IBS SC relative position.

The results show that a reduction in accuracy of control to 0.25 m enables to reduce propellant consumption 5.7 times at almost same OB deorbiting rate. At an accuracy less than 0.5 m the propellant consumption decreases 13.5 times as compared with the first case, whereas the deorbiting rate slows down by 1% only. Softening of requirements for accuracy of control to 0.75 m leads to a 27.7 times decrease in propellant consumption, while the deorbiting rate falls by 6% as compared with the first case. Hence, for the mentioned input data, the rational accuracy of control ranges from 0.5 to 0.75 m, since further softening of the requirements entails a significant decrease in the OB deorbiting rate.

CONCLUSIONS

The results of synthesis and analysis of system for control over IBS SC motion with respect to OB have been presented. The system has been shown to ensure required accuracy of control over relative motion of IBS SC considering specific effects transmitted by ion beam, external disturbances, inaccuracy of relative position measurement, and non-ideality of reactive actuators. The proposed method for synthesis of controller enables to ensure a compromise between robust stability, performance, and control costs. Robust stability and robust performance of the system with respect to varying parameters of the plant have been confirmed both by formal criterion and by computer simulation with the use of nonlinear mathematical model with a wide range of orbital disturbances acting on the mentioned system taken into account. A rational softening of the requirements for accuracy of control has been shown to essentially reduce propellant consumption for maintaining the IBS SC relative position while keeping a sufficient OB deorbiting rate.

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СИСТЕМА КЕРУВАННЯ ВІДНОСНИМ РУХОМ КОСМІЧНОГО АПАРАТУ ДЛЯ БЕЗКОНТАКТНОГО ВИДАЛЕННЯ КОСМІЧНОГО СМІТТЯ

Вступ. Розглянуто питання створення системи керування космічного апарату для безконтактного видалення космічного сміття з використанням технології «Пастух з іонним променем». Ця система необхідна для того, щоб забезпечити умови ефективної передачі іонним променем гальмуючого імпульсу об'єкту космічного сміття в фазі відведення.

Проблематика. При синтезі та аналізі системи необхідно враховувати вплив іонного променя, широкий спектр орбітальних збурень, неточності визначення відносного положення та реалізації керуючих впливів, нестаціонарність і параметричну невизначеність об'єкта керування, а також обмеження на керування.

Мета. Синтез системи керування відносним рухом космічного апарату для безконтактного видалення космічного сміття.

Матеріали й методи. Для синтезу системи використано метод змішаної чутливості. Вимоги до регулятора задано в частотній області за допомогою обраних вагових функцій. Аналіз робастності системи виконано на базі методології структурованих сингулярних чисел.

Результати. Робастність системи та відповідність заданим вимогам підтверджено як за допомогою формального критерію, так і шляхом комп'ютерного моделювання. Показано, що раціональне зниження вимог до точності керування дозволяє істотно знизити витрату робочого тіла на підтримку відносного положення при збереженні прийнятної швидкості відведення космічного сміття.

Висновки. Синтезовано систему керування, яка забезпечує необхідний компроміс між робастною стійкістю, якістю і витратами на керування з урахуванням широкого спектра розглянутих збурень.

Ключові слова: система керування, відносний рух, космічне сміття, концепція «Пастух з іонним променем», робастна стійкість.

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СИСТЕМА УПРАВЛЕНИЯ ОТНОСИТЕЛЬНЫМ ДВИЖЕНИЕМ КОСМИЧЕСКОГО АППАРАТА ДЛЯ БЕСКОНТАКТНОГО УДАЛЕНИЯ КОСМИЧЕСКОГО МУСОРА

Введение. Рассмотрены вопросы создания системы управления космического аппарата для безконтактного удаления космического мусора с использованием технологии «Пастух с ионным лучом». Эта система необходима для того, чтобы обеспечить условия эффективной передачи ионным лучом тормозящего импульса объекту космического мусора в фазе увода.

Проблематика. При синтезе и анализе системы необходимо учитывать воздействия ионного луча, широкий спектр орбитальных возмущений, неточности определения относительного положения и реализации управляющих воздействий, не стационарность и параметрическую неопределенность объекта управления, а также ограничения на управляющие воздействия.

Цель. Синтез системы управления относительным движением космического аппарата для бесконтактного удаления космического мусора.

Материалы и методы. Для синтеза системы использован метод смешанной чувствительности. Требования к регулятору заданы в частотной области с помощью выбранных весовых функций. Анализ робастности системы выполнен на базе методологии структурированных сингулярных чисел.

Результаты. Робастность системы и соответствие заданным требованиям подтверждены как с помощью формального критерия, так и путем компьютерного моделирования. Показано, что рациональное снижение требований по точности управления позволяет существенно снизить расход рабочего тела на поддержание относительного положения при сохранении приемлемой скорости увода космического мусора.

Выводы. Синтезирована система управления, которая обеспечивает необходимый компромисс между робастной устойчивостью, качеством и затратами на управление с учетом широкого спектра рассмотренных возмущений.

Ключевые слова: система управления, относительное движение, космический мусор, концепция «Пастух с ионным лучом», робастная устойчивость.