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# MATHEMATICAL MODELING OF CYLINDRICAL ADAPTER DYNAMICS UNDER THE ACTION OF LOCAL MOMENTARY LOADS



Methods and computing program to determine displacements and accelerations of points of the cylindrical shell (adapter) middle surface under the action of local momentary loads have been developed. Effect of local load on the oscillation parameters and deformation of the shell (adapter) has been studied by test example. The displacement and acceleration under the action of local momentary load have been established to be localized in the action points.

The designed methods can apply to initial calculations of the parameters of oscillations and deformations of cylindrical shell (adapter) structural elements undergoing the action of heavy local momentary loads in the course of operation.

Keywords: cylindrical shell, adapter, the local momentary load, and distribution of accelerations.

The stage separation of the launch vehicle and space equipment using charge-driven piston mechanisms and flexible linear shaped charges located in the appropriate adapters is accompanied with heavy local momentary loads, which can lead to breakdowns. The plastic deformation zones in the separated elements are known to be comparable with the thickness of the separated elements, with other elements of critical equipment operating in the elastic range. This allows us to use the theory of elastic shells for assessing operational status of these elements. To this end, it is necessary to determine the stress distribution and the nature of oscillations occurring in cylindrical adapters under the action of local momentary loads of high intensity, to formulate appropriate boundary problems, to create or to improve the existing methods and software, to estimate the strain state and accelerations, and to justify the reliability of the results obtained.

Methods for solving boundary problems of oscillation and stress-strain state of smooth and ribbed shells on elastic foundation under unsteady loads using numerical integration methods and finite differences have been developed in [1, 2]. Software to determine the nature of oscillations and the stress-strain state of heterogeneous shells of revolution under distributed momentary loads has been proposed in [3, 4]. However, they are unsuitable for evaluating the oscillations and mechanical effects of charge-driven piston mechanisms on the adapters, insofar as the existing versions of the software are not designed for numerical study of local action of momentary loads on the mentioned elements. Since the cylindrical adapters undergo significant local momentary loads while the stages of launch vehicles and space equipment are separating, it is necessary to identify the mechanical effect of the separated parts on the operating capacity of space equipment. Given this, the dynamics of cylindrical shell as adapter under the action of local high-intensity pulses generated by explosions of charge-driven piston mechanisms will be mathematically simulated. In this regard, this research has improved the existing methods for solving the problems of dynamics of shells of revolution and appropriate software for their use in the calculation of adapters.

The object of this research is cylindrical adapter, the size and properties of material of which and conditions of local momentary loads are provided by *Pivdenne* Design Office. Therefore, it is necessary to formulate a respective problem of adapter dynamics and on the basis of its solution to conclude on the level of displacements and accelerations and potential emergency situations.

### THE STATEMENT OF THE PROBLEM. BASIC EQUATIONS

Hereafter, the problem of determining the strain state of closed cylindrical shell (adapter) under the action of local momentary loads distributed over its surface is considered. At the ends of the shell, the boundary conditions of hinge support are established.

The solution of the problem is based on classical theory of shells [5] and the energy method. This approach has been described in detail and used to solve the problems of dynamic membrane under the action of axially symmetric momentary load uniformly distributed along its length [4]. The variation equation of motion obtained in that research can be used for the local loads as well. For shell without supports, the variation equation is as follows:

$$\begin{split} & \int_{t_1}^{t_2} \int_{0}^{2\pi} \int_{0}^{L/r} \left[ \left( \frac{Eh}{1-\mu^2} L_{\delta u} - r^2 q_1 \right) \delta u + \left( \frac{Eh}{1-\mu^2} L_{\delta v} - r^2 q_2 \right) \times \\ & \times \delta v + \left( \frac{Eh}{1-\mu^2} L_{\delta w} - r^2 q_3 \right) \delta w \right] d\xi d\theta dt = 0, \end{split} \tag{1}$$

where

$$L_{\delta u} = \frac{\partial^2 u}{\partial \xi^2} + \frac{1 - \mu}{2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1 + \mu}{2} \frac{\partial^2 v}{\partial \xi \partial \theta} - \mu \frac{\partial w}{\partial \xi} - \sigma_0 \frac{\partial^2 u}{\partial \tau^2};$$

$$L_{\delta v} = \frac{1+\mu}{2} \frac{\partial^2 u}{\partial \xi \partial \theta} + (1+a^2) \frac{\partial^2 v}{\partial \theta^2} + \frac{1-\mu}{2} (1+4a^2) \frac{\partial^2 v}{\partial \xi^2} -$$

$$\begin{split} &-\frac{\partial \mathbf{w}}{\partial \theta} + \mathbf{a}^2 \bigg[ (2-\mu) \, \frac{\partial^3 \mathbf{w}}{\partial \xi^2 \, \partial \theta} + \frac{\partial^3 \mathbf{w}}{\partial \theta^3} \, \bigg] - \sigma_0 \, \frac{\partial^2 \mathbf{v}}{\partial \tau^2} \, ; \\ \mathbf{L}_{\delta \mathbf{w}} &= \mu \, \frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \theta} - a^2 \bigg[ (2-\mu) \, \frac{\partial^3 \mathbf{v}}{\partial \xi^2 \, \partial \theta} + \frac{\partial^3 \mathbf{v}}{\partial \theta^3} \, \bigg] - \mathbf{w} - \\ &- a^2 \, \bigg( \frac{\partial^4 \mathbf{w}}{\partial \xi^4} + 2 \, \frac{\partial^4 \mathbf{w}}{\partial \xi^2 \, \partial \theta^2} + \frac{\partial^4 \mathbf{w}}{\partial \theta^4} \, \bigg) - \sigma_0 \, \frac{\partial^2 \mathbf{w}}{\partial \tau^2} . \end{split}$$

Here, E,  $\rho_0$ ,  $\mu$  are elasticity modulus, density, and Poisson coefficient of the shell material; u, v, w are longitudinal, circular, and normal displacements of the shell middle surface;  $x = r\xi$ ,  $y = r\theta$ ,  $z = r\zeta$  are longitudinal, circular, and normal coordinates;  $\tau = t/T$  is dimensionless time;  $T = t_2 - t_1$  is certain time interval;  $t_1$ ,  $t_2$  are fixed time points; h, r, L are thickness, radius of middle surface and length of the shell, respectively;  $a^2 = h^2/(12r^2)$ ;

$$\begin{split} &\sigma_0 = \frac{\left(1 - \mu^2\right)\rho_0\, r^2}{ET^{\,2}}; \; q_1 = q_{01} \cdot \overline{q}_1 \left(\xi,\;\theta\right) \cdot \overline{q}_{1t} \left(\tau\right); \\ &q_2 = q_{02} \cdot \overline{q}_2 \left(\xi,\;\theta\right) \cdot \overline{q}_{2t} \left(\tau\right); \; q_3 = q_{03} \cdot \overline{q}_3 \left(\xi,\;\theta\right) \cdot \overline{q}_{3t} \left(\tau\right); \\ &q_{01},\; q_{02},\; q_{03} \; \text{are maximum values of the external load components; } \overline{q}_1,\; \overline{q}_2,\; \overline{q}_3 \; \text{are dimensionless functions describing changes in external load depending on spatial coordinates; } \overline{q}_{1t},\; \overline{q}_{2t},\; \overline{q}_{3t} \; \text{are dimensionless functions showing time dependence} \end{split}$$

In this case the spatial functions are set as single stages:  $\overline{\mathbf{q}}_{i} = 1$ , if  $\xi_{1i} \leq \xi \leq \xi_{2i}$ ,  $\theta_{1i} \leq \theta \leq \theta_{2i}$  and is equal to zero  $\overline{\mathbf{q}}_{i} = 0$ , in other points of the surface.  $\xi_{1i}$ ,  $\xi_{2i}$ ,  $\theta_{1i}$ ,  $\theta_{2i}$  are coordinates of *i*-th local load. The load changes with time as:  $\overline{\mathbf{q}}_{ti}(\boldsymbol{\tau}) = \alpha_{1i} + \alpha_{2i} \boldsymbol{\tau}$  at  $\tau_{1i} \leq \tau \leq \tau_{2i}$ ;  $\overline{\mathbf{q}}_{ti}(\boldsymbol{\tau}) = 0$  in other time points;  $\alpha_{1i}$ ,  $\alpha_{2i}$  are dimensionless coefficients.

In order to solve the equations (1) the method of eigen mode expansion is used.

### **RESEARCH METHODS**

The displacements of points of middle surface are approximated with double trigonometric series by spatial coordinates

$$\begin{split} u &= \sum\limits_{m=1}^{M} \sum\limits_{n=0}^{N} (u_{1mn}^{1}(\tau) \cos n\theta + u_{1mn}^{2}(\tau) \sin n\theta) \cos d_{m} \xi; \\ v &= \sum\limits_{m=1}^{M} \sum\limits_{n=0}^{N} (u_{2mn}^{1}(\tau) \sin n\theta + u_{2mn}^{2}(\tau) \cos n\theta) \sin d_{m} \xi; \end{split}$$

of the load.

$$W = \sum_{m=1}^{M} \sum_{n=0}^{N} (u_{3mn}^{1}(\tau)\cos n\theta + u_{3mn}^{2}(\tau)\sin n\theta) \sin d_{m}\xi, \quad (2)$$

where  $u_{1mn}^1(\tau)$ ,  $u_{2mn}^1(\tau)$ ,  $u_{3mn}^1(\tau)$  are searched time functions;  $d_m = m\pi r/L$ .

Having put (2) to (1), differentiated, and integrated by  $\xi$ ,  $\theta$  coordinates for independence and arbitrary character of displacements one can get the following systems of ordinary non-uniform differential equations:

$$\mathbf{M}^{1} \frac{\partial^{2} \mathbf{u}^{1}}{\partial \tau^{2}} + \mathbf{S}^{1} \mathbf{u}^{1} = \mathbf{Q}^{1}, \quad \mathbf{M}^{2} \frac{\partial^{2} \mathbf{u}^{2}}{\partial \tau^{2}} + \mathbf{S}^{2} \mathbf{u}^{2} = \mathbf{Q}^{2}. \quad (3)$$

Here  $\mathbf{M}^1$ ,  $\mathbf{M}^2$ ,  $\mathbf{S}^1$ ,  $\mathbf{S}^2$  are mass and rigidity matrixes whose elements depend on the shell parameters and the wave formation parameters m, n. The superscripts 1, 2 show that when compiling the mass and the rigidity matrixes, the first (superscript 1) and the second (superscript 2) summands, respectively, are used in the expressions (2);  $\mathbf{u}^1$ ,  $\mathbf{u}^2$  are column vector of time functions  $u^1_{1mn}$ ,  $u^1_{2mn}$ ,  $u^1_{3mn}$  and  $u^2_{1mn}$ ,  $u^2_{2mn}$ ,  $u^2_{3mn}$ , respectively;  $\mathbf{Q}^1$ ,  $\mathbf{Q}^2$  are column vectors obtained from the integration by spatial coordinates of external loads.

For applying the eigen mode method [6], at the first stage, the eigenvalue problem is considered. Let us find the diagonal matrixes  $\mathbf{P}^1$ ,  $\mathbf{P}^2$ , whose elements are squared eigenvalues  $(p_k^1)^2$ ,  $(p_k^2)^2$ , and eigen mode matrixes  $\mathbf{A}^1$ ,  $\mathbf{A}^2$ , whose elements are arbitrary eigen mode constants  $a_k^1$ ,  $a_k^2$ .

Further, let us transform the equations (3) reducing them to the normal coordinates and normalizing the eigen mode matrixes with respect to the mass matrix. In this case, the mass matrixes in normal coordinates are single matrixes, while the rigidity matrixes are diagonal matrixes whose elements are squared eigenvalues  $(p_k^1)^2$ ,  $(p_k^2)^2$ , respectively. To the right, there are the elements of vectors  $\mathbf{Q}_g^1 = (\overline{\mathbf{A}}^1)^T \mathbf{Q}^1$ ,  $\mathbf{Q}_g^2 = (\overline{\mathbf{A}}^2)^T \mathbf{Q}^2$ .

As a result, instead of the systems (3), one can get a series of independent equations:

$$\ddot{u}_{g,i}^{1}(t) + (p_{i}^{1})^{2} u_{g,i}^{1}(t) = q_{g,i}^{1}(t);$$

$$\ddot{u}_{g,i}^{2}(t) + (p_{i}^{2})^{2} u_{g,i}^{2}(t) = q_{g,i}^{2}(t),$$
(4)

whose number is equal to the number of members in the series (2).

If the energy dissipation as a result of shell oscillations is approximately taken into account according to [6], one can get the following equations instead of (4)

$$\ddot{u}_{g,i}^{1}(t) + 2c_{g,i}^{1} \dot{u}_{g,i}^{1}(t) + (p_{i}^{1})^{2} u_{g,i}^{1}(t) = q_{g,i}^{1}(t);$$

$$\ddot{u}_{g,i}^{2}(t) + 2c_{g,i}^{2} \dot{u}_{g,i}^{2}(t) + (p_{i}^{2})^{2} u_{g,i}^{2}(t) = q_{g,i}^{2}(t), \quad (5)$$

where  $c_{g,i}^1 = \gamma_i p_i^1$ ,  $c_{g,i}^2 = \gamma_i p_i^2$  are damping constants by *i*-th eigen mode,  $\gamma_i$  are respective damping coefficients.

Having obtained solutions of equations (4), (5), let us come back to the initial system of searched functions using the expressions

$$\mathbf{u}^1 = \overline{\mathbf{A}}^1 \mathbf{u}_o^1, \mathbf{u}^2 = \overline{\mathbf{A}}^2 \mathbf{u}_o^2.$$

It should be noted that to solve the problem of dynamics of smooth hinge-supported cylindrical shell it is not really necessary to use the method of eigen mode expansion, inasmuch as the members of the series (2) are the eigen modes in the case of such shell. This research includes verification of the method for the case of local momentary loads in order to expand its use over computing the dynamics of shells with various complications such as stiffeners, attached weights, elastic foundations, etc. The shell is assumed to be at rest before the action of external load.

The solution of equations (5) can be found using the Duhamel integral [6]. Under the zero initial conditions, for each *i*-th mode (superscripts are not indicated) it is as follows:

$$\begin{array}{c} \text{txes} \\ \text{hile} \\ \text{nose} \\ \text{re-} \\ \text{s of} \\ \end{array} \begin{array}{c} u_{\mathrm{g}}(t) = \frac{q_{\mathrm{g}}}{p_{\mathrm{g}}^{2}} \left\langle \alpha_{1} + \alpha_{2} \left(\tau - \frac{2\gamma}{p}\right) - \mathrm{e}^{-\gamma p(\tau - \tau_{1})} \left\{ \left[\alpha_{1}\beta_{1} + \frac{2\gamma}{p}\right] + \left[\alpha_{1} + \alpha_{2} \left(\tau_{1} - \frac{2\gamma}{p}\right)\right] \times \right. \\ \times \cos p_{\mathrm{d}} \left(\tau - \tau_{1}\right) + \left[\alpha_{1} + \alpha_{2} \left(\tau_{1} - \frac{2\gamma}{p}\right)\right] \times \\ \times \cos p_{\mathrm{d}} \left(\tau - \tau_{1}\right) \right\} \left\langle \left[\alpha_{1}\beta_{1} + \alpha_{2} \left(\beta_{1}\tau_{1} + \frac{\beta_{2}}{p}\right)\right] \times \\ \times \sin p_{\mathrm{d}} \left(\tau - \tau_{1}\right) + \left[\alpha_{1} + \alpha_{2} \left(\tau_{1} - \frac{2\gamma}{p}\right)\right] \cos p_{\mathrm{d}} \left(\tau - \tau_{1}\right) \right\} + \\ \left. + \mathrm{e}^{-\gamma p(\tau - \tau_{2})} \left\{ \left[\alpha_{1}\beta_{1} + \alpha_{2} \left(\beta_{1}\tau_{2} + \frac{\beta_{2}}{p}\right)\right] \sin p_{\mathrm{d}} \left(\tau - \tau_{2}\right) + \right. \end{array} \tag{6}$$

$$\begin{split} &+\left[\alpha_{1}+\alpha_{2}\left(\tau_{2}-\frac{2\gamma}{p}\right)\right]\cos p_{d}\left(\tau-\tau_{2}\right)\right\}\right\rangle, \quad \text{is value } \tau > \tau_{2} \\ &\text{where } p_{d}=p\sqrt{1-\gamma^{2}}, \, \beta_{1}=\frac{\gamma}{\sqrt{1-\gamma^{2}}}, \, \beta_{2}=\frac{1-2\gamma^{2}}{\sqrt{1-\gamma^{2}}}. \end{split}$$

It should be noted that if in (6) it is assumed that  $\gamma=0$ , one can obtain the solution of equations (4). General multiplier in expressions (6), namely  $\frac{q_g}{p^2}$ , is the solution of static problem.

To find he acceleration of points of the shell middle surface let us take the second derivative from the expressions (6) by variable  $\tau$  and obtain:

$$\begin{split} \ddot{u}_{g}(\tau) &= q_{g} e^{-\gamma p(\tau-\tau_{i})} \left[ c_{3} sin \; p_{d}(\tau-\tau_{i}) + c_{4} cos \; p_{d}(\tau-\tau_{i}) \right] \\ &\quad is \; value \; \tau_{i} \leq \tau \leq \tau_{3}; \end{split}$$

$$\begin{split} \ddot{u}_{g}(\tau) &= q_{g} \{ e^{-\gamma p(\tau-\tau_{1})} \left[ c_{3} sin \ p_{d}(\tau-\tau_{_{1}}) + c_{4} cos \ p_{d}(\tau-\tau_{_{1}}) \right] - \\ &- e^{-\gamma p(\tau-\tau_{2})} \left[ d_{3} sin \ p_{d}(\tau-\tau_{_{2}}) + d_{_{4}} cos \ p_{_{d}}(\tau-\tau_{_{2}}) \right] \end{split}$$

is value  $\tau > \tau$ ,

$$\begin{split} &\text{where } c_3^{} = (1 - 2 \gamma^2) \, b_1^{} - 2 \gamma \, \sqrt{1 - \gamma^2} \, b_2^{}; c_4^{} = (1 - 2 \gamma^2) \, b_2^{} - \\ &- 2 \gamma \, \sqrt{1 - \gamma^2} \, b_1^{}; \ d_3^{} = (1 - 2 \gamma^2) \, b_3^{} - 2 \gamma \, \sqrt{1 - \gamma^2} \, b_4^{}; \\ d_4^{} = (1 - 2 \gamma^2) \, b_4^{} + 2 \gamma \, \sqrt{1 - \gamma^2} \, b_3^{}; \ b_1^{} = \left[\alpha_1 \beta_1^{} + \\ &+ \alpha_2^{}(\beta_1 \tau_1^{} + \frac{\beta_2^{}}{p})\right]; \ b_2^{} = \left[\alpha_1^{} + \alpha_2^{} \, \left(\tau_1^{} - \frac{2 \gamma^{}}{p}\right)\right]; \\ b_3^{} = \left[\alpha_1^{} \beta_1^{} + \alpha_2^{}(\beta_1 \tau_2^{} + \frac{\beta_2^{}}{p})\right]; \ b_4^{} = \left[\alpha_1^{} + \alpha_2^{} \, \left(\tau_2^{} - \frac{2 \gamma^{}}{p}\right)\right]. \end{split}$$

## EXAMPLE OF COMPUTATION OF STRAIN STATE PARAMETERS AND SHELL (ADAPTER) OSCILLATIONS UNDER THE ACTION OF LOCAL MOMENTARY LOADS

For the smooth cylindrical shell (adapter) with parameters  $\rho = 2.65 \cdot 10^3$  kg/m³;  $E = 0.7 \times 10^{11}$  N/m²;  $\mu = 0.3$ ; r = 0.185 m; L = 0.27 m; h = 0.0045 m the bend reflections w and accelerations  $\ddot{w}$  of the points of shell middle surface under the action of the following loads have been calculated: within time from zero to 0.007 s the shell undergoes the action of two normal (to the middle surface) forces distributed over the rectangular areas having size  $a \times a = 0.034$   $m \times 0.034$  m, their centers have the coordinates  $\xi_1 = \xi_3 = 0.2$ ;  $\theta_1 = \pi/5$  (point 1) and  $\theta_3 = 19\pi/15$  (point 3). Within time  $0 \le t < 0.007$  the load does not vary with time:  $q_1 = q_2 = q_{max}$ ,

and is equal to zero  $q_1 = q_2 = 0$  at  $t \ge 0.007$ . At time  $0.7 \le t < 0.707$  two more forces of the same magnitude and direction apply to the shell in the points with coordinates  $\xi_2 = \xi_4 = 0.2$ ;  $\theta_2 = 11\pi/15$  (point 2) and  $\theta_4 = 9\pi/5$  (point 4).

The calculations are made for keeping the members in series (2) up to M = 30, N = 40, when a convergence of the results is achieved. As the number of members grows the quantitative values het more accurate, but the behavior of curves remains the same.

Insofar as for determining the parameters of strain state of the shell, the Eigen mode expansion method is used, the Table contains the results of calculation of the lower eigen frequencies:

One can see form the Table that the minimum frequency is reported for m=1, n=5 and is equal to 1132 Hz. The corresponding maximum eigen period  $t_{\rm max} \approx 0.00088$  s, is significantly less than time of load action and, moreover, less than time elapsing between the application of the first and the second groups of forces. This fact gives reason for stating that there are no resonance effects as a result of the action of the above mentioned forces on the shell.

Fig. 1 shows time dependences of dimensionless bend deflection in the point of application of force (*solid line*) and in the point with coordinates  $\theta = \pi/5$ ;  $\xi = 0.5$  (*dashed line*). The calculations are made for time interval  $t = 0 \div 0.01$  s at a pitch of 0.0001 s.

One can see from Fig.1 that the shell displacement reaches maximum magnitude during the action of load. After relief, the points of shell middle surface oscillate about the equilibrium position in free (unloaded) state. The maximum deflection in

Eigen Frequencies of Shell Oscillations, Hz

m	n						
	2	3	4	5	6	7	8
1	2204	1477	1139	1132	1361	1733	2201
2	3641	3034	2559	2294	2265	2451	2804
3	4274	3955	3659	3462	3411	3525	3796

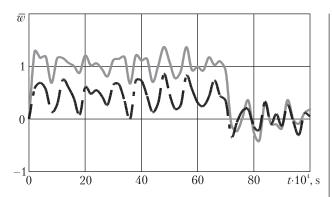
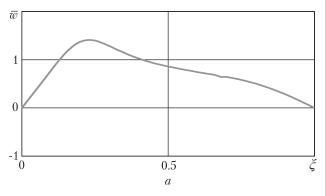


Fig. 1. Time dependence of bend deflections of the shell points



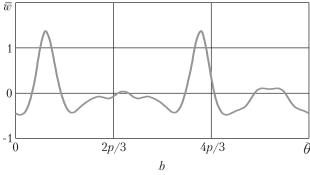


Fig. 2. Curves of bend deflections of the shell points in cross sections  $\theta = \pi/5$  (a) and  $\xi = 0.5$  (b)

point 1 under the action of forces exceeds that after the relief little bit more than trice.

Fig. 2 features the strain shape of the shell at time t = 0.0048 s in cross sections  $\theta = \pi/5$  (Fig. 2a) and  $\xi = 0.2$  (Fig. 2b). To obtain dimensional bend deflections of the shell one can use the formula  $w = \frac{q_{\text{max}} (1-v^2)r^2}{Eh} \overline{w}. \text{ A force pulse of } 2.18 \text{ N} \cdot \text{s corres}$ 

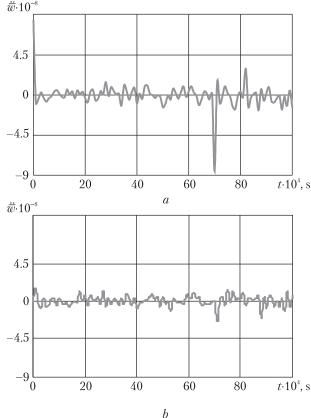


Fig. 3. Time dependence of acceleration of the shell points

ponds to the load  $q_{max}$  =  $2.694 \cdot 10^5 \, N/m^2$  acting on the shell and distributed over the given load areas, with maximum deflection of the shell surface from the equilibrium position reaching  $\sim 0.37 \cdot 10^{-4}$  M.

Fig. 3 shows the calculated accelerations of shell middle surface points within time interval  $t = 0 \div 0.01$  s at a pitch of 0.0001 s. The curve on Fig. 3a describes acceleration in point 1, that on Fig. 3b shows acceleration in the point with coordinates  $\theta = \pi/5$ ;  $\xi = 0.5$ .

The results show that the maximum acceleration is reported in the points of application of external forces at the beginning and at the end of their action. At other time, the acceleration is less by almost an order of magnitude. Let us calculate dimensional acceleration by the formula

 $\ddot{w} = \frac{q_{max} (1-v^2)r^2}{Fh T^2} \ddot{\overline{w}}$ . Having made the calculations,

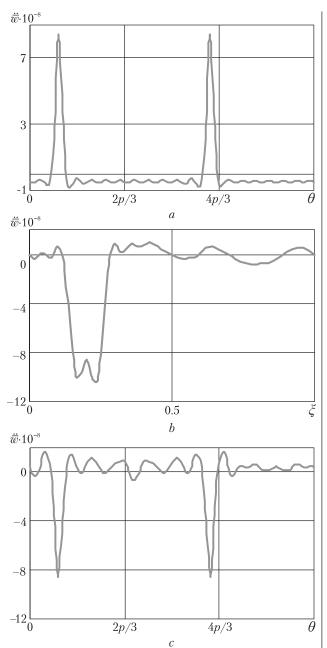


Fig. 4. Acceleration distribution over the shell surface at the beginning and at the end of load action

one can get that, for example, at time t=0 the absolute acceleration of point 1 is equal to  $\ddot{w}=2241G$ , at time t=0.007 s it reaches  $\ddot{w}=2277G$ , at time t=0.0082 s it is equal to  $\ddot{w}=788G$ , that in the point with coordinates  $\theta=\pi/5$ ;  $\xi=0.5$  at time

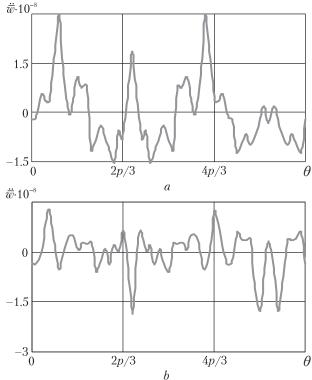


Fig. 5. Acceleration distribution over the shell surface after relief

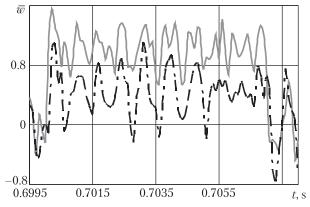


Fig. 6. Time dependence of bend deflections of shell points under the action of the second pair of forces

t = 0.001 s is equal to  $\ddot{w} = 396G$ , at time t = 0.0071 s it reaches  $\ddot{w} = 653G$ . It should be noted that the calculated acceleration (except for those in point 1 at t = 0 and at t = 0.007 s) is close to the experiment data measured after the relief.

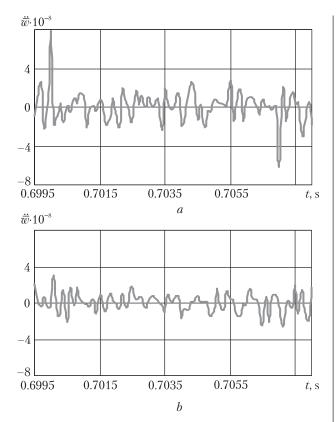


Fig. 7. Time dependence of acceleration of shell points under the action of the second pair of forces

Fig. 4 feature acceleration distribution over the shell surface at time t=0 in the cross section  $\xi=0.2$  (Fig. 4a), and at time t=0.007 s in the cross sections  $\theta=\pi/5$  (Fig. 4b) and  $\xi=0.2$  (Fig. 4c). As one can see from Figs, the maximum acceleration of points of shell middle surface at the beginning and at the end of external load action is localized in the action points.

At other time, the peak acceleration of points of shell middle surface appears in various places as showed in Fig. 5*a* ( $\xi$  = 0.2) and in Fig. 5*b* ( $\xi$  = 0.5), at t = 0.0082 s.

Further, time of action of forces applied in points 2 and 4 (0.7  $\leq$  t < 0.707) is considered. Fig. 6 features the time dependences of shell bend deflection. The solid curve shows the time dependence of dimensionless bend deflection in the action point 2 ( $\theta$  = 11 $\pi$ /15;  $\xi$  = 0.2), the dashed

curve shows that in the point with coordinates  $\theta = 11\pi/15$ ;  $\xi = 0.5$ . Like in the case of the first pair of forces, at  $0.7 \le t < 0.707$  the shell oscillates around the equilibrium position as a result of the action. The maximum bend deflection (at a force pulse of  $2.18 \text{ n} \cdot \text{s}$ ) reaches  $\sim 0.41 \cdot 10^{-4} \text{ m}$ . This value is little bit higher than that under the action of the first pair of forces, as a result of interference of oscillations caused by the action of all forces. It should be noted that the maximum dimensional bend deflection of the shell does not exceed 1 percent of its thickness, which testifies to elastic deformation of the shell.

Fig. 7 shows the time dependence of acceleration within  $0.6995 \le t < 0.708$ , under the action of the second pair of forces. Like under the case of the first pair of forces, the maximum acceleration appears in the action points at the beginning and at the end of action. The maximum dimensional acceleration is equal to  $\ddot{w} = 2097 G$  at t = 0.7 s, and  $\ddot{w} = 1657 G$  at t = 0.707 s.

### CONCLUSIONS

A computation pattern, a method, and a program for calculating displacement and acceleration of points of middle surface of the hinge-supported shell under the action of local momentary load have been designed for simulating the dynamics of cylindrical shell structural elements. Effect of local load on the oscillation parameters and deformation of the shell (adapter) has been studied by test example. The displacement and acceleration under the action of local momentary load have been established to be localized in the action points. The comparison with the experimental data provided by *Pivdenne* Design Office has showed that neither large deformations nor resonance effects that can lead to breakdown are expected for the given adapter and the type of mechanical action of charge-driven piston mechanisms. At the same time, insofar as the calculated maximum acceleration exceeds the experimental ones twice, this fact should be taken into consideration when designing the configuration of fastenings.

The designed method can be used for initial calculations of the parameters of oscillations and deformations of cylindrical shell (adapter) structural elements undergoing the action of heavy local momentary loads in the course of operation.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ДИНАМІКИ ЦИЛІНДРИЧНОГО АДАПТЕРА ПІД ДІЄЮ ЛОКАЛЬНОГО ІМПУЛЬСНОГО НАВАНТАЖЕННЯ

Розроблена методика та обчислювальна програма для визначення переміщень і прискорень точок серединної поверхні циліндричної оболонки— адаптера— під дією

локальних короткочасних навантажень. На тестовому прикладі проведені дослідження впливу локального навантаження на параметри коливань і деформування оболонки. Показано, що при локальних і короткочасних навантаженнях спостерігається локалізація максимальних переміщень і прискорень в місцях прикладення зовнішніх сил.

Розроблена методика може бути використана для попередніх обрахунків параметрів коливань і деформування елементів конструкцій у вигляді гладких циліндричних оболонок під дією локальних динамічних навантажень великої інтенсивності.

*Ключові слова*: циліндрична оболонка, адаптер, локальне імпульсне навантаження, розподіл прискорень.

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### МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДИНАМИКИ ЦИЛИНДРИЧЕСКОГО АДАПТЕРА ПРИ ДЕЙСТВИИ ЛОКАЛЬНОГО ИМПУЛЬСНОГО НАГРУЖЕНИЯ

Разработана методика и вычислительная программа для определения перемещений и ускорений точек срединной поверхности цилиндрической оболочки — адаптера — под действием локальных кратковременных нагрузок. На тестовом примере проведены исследования влияния локального кратковременного нагружения на параметры колебаний и деформирования оболочки. Показано, что при локальных и кратковременных нагрузках имеет место локализация максимальных перемещений и ускорений в местах приложения внешних сил. Разработанная методика может быть использована для предварительных расчетов колебаний и параметров деформирования элементов конструкций в виде гладких цилиндрических оболочек при действии локальных динамических нагрузок большой интенсивности.

*Ключевые слова*: цилиндрическая оболочка, адаптер, локальное импульсное нагружение, распределение ускорений.

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