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PREDICTION OF POLLUTION LEVEL BETWEEN MEASUREMENT POINTS BY MATHEMATICAL MODELING USING INTERPOLATION AND RECURSION

Introduction. *There are different mathematical modeling methods for assessing riverbed pollution consequences. Among them, there is the series of Harvey Jobson's empirical hydrological equations, which allows measurements between a spill point and a measurement point. The measurement between two necessary points located consecutively along the riverbed is made by two calculations for these points and results comparison. Is makes it possible to predict the dynamics of the riverbed pollution spread in time and space.*

Problem Statement. *The forecast by two points is, likely, inconclusive because the result is an array of insufficient size. Thus, there is a need to expand a result dataset to improve understanding the accuracy of calculations of pollution spread dynamics.*

Purpose. *The purpose is to develop a method that determines pollution level between certain measurement points and does not require additional data. The output data for the array are the peak concentration and the time for the concentration to reach its peak at a particular point in the body of water.*

Material and Methods. *The mathematical method is based on the interpolation results in combination with the recursion method, as a result. The output data of the method are the concentration peak and the time for the concentration to reach its peak.*

Results. *In this research, the method has been developed and validated through a series of simulations. The method validity has been proven by the graphical method, with graphs similarity as main criterion.*

Conclusions. *Using the method together with a series of Harvey Jobson's empirical hydrological equations as part of intelligent processing of the results makes it possible to predict pollution level between specific riverbed points, by mathematical modeling, with the help of interpolation and recursion methods and thereby to increase emergency consequences prediction accuracy.*

Keywords: water quality assessment, riverbed, water pollution consequences forecasting, mathematical modeling, interpolation, and recursion.

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There are different mathematical modeling methods for assessing riverbed pollution consequences. Among them, there is the series of Harvey Jobson's empirical hydrological equations, which allows measurements between a spill point and a required measurement point. The output data of the method are the concentration peak and the time for the concentration to reach its peak.

Predicting the results by two points is likely to be unconvincing because the result is an array of insufficient size. This problem is especially relevant if the measured points are at a great distance from each other.

Thus, the purpose of this research is to expand the array of results for improving the understanding of the pollution spread dynamics.

Fick's second law modified by Taylor can be used to determine gradient values dynamics,

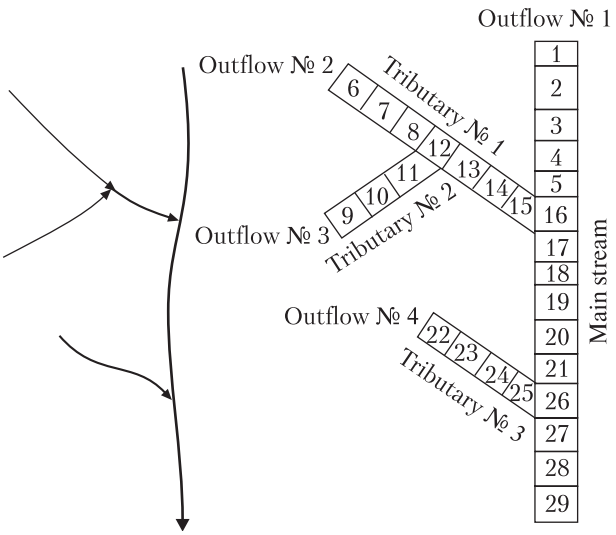

Fig. 1. Riverbed as a series of sections

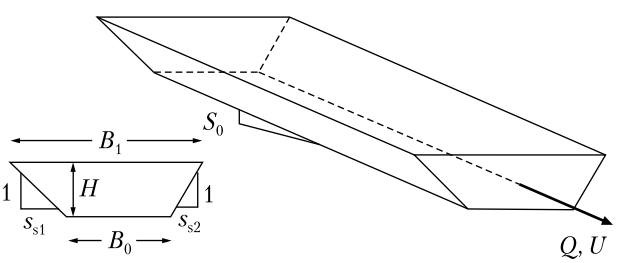

Fig. 2. Trapezoidal channel model

which requires more detailed riverbed data, the function of which is used to characterize the riverbed itself.

The task is to develop a method that determines the results of pollution between defined measurement points and to create an array of the values. The result for the measurement point is the peak concentration and the time for the concentration to reach its peak.

The existing solution for this problem has been implemented in such software models as QUAL2K [3, 4], WASP8 [5], Visual Plumes [6], etc.

The principle underlying the QUAL2K model is to consider the riverbed as a series of proportional segments. Each section has constant hydrological characteristics that simulate the process of migration of substances in water bodies. The river can be considered both with and without tributaries [7]. The scheme is shown in Fig. 1.

The section corresponds to a trapezoidal channel with constant properties.

WASP8 considers a riverbed to be a collection of channels. In addition, the section is divided into six different types, or "fields" that describe the migration of water bodies at different depths, from surface water to sediments. Also, if the channel is wide, it may be considered a set of segments located in parallel while modeling their sequence in the stream. An example scheme showing how a wide riverbed is branched into 3 sets of segments and then merged is shown in Fig. 3.

The Visual Plumes model considers the river to be a three-dimensional plume described by static hydrological parameters in a static or dynamic state, depending on the model used. The plume is also a set of sequences of elements.

Also, the following Fick's second law of diffusion formula derived by Taylor can be used, which allows creating a substance concentration gradient in the riverbed:

$$\frac{dC}{dt} + u\frac{dC}{dx} + v\frac{dC}{dy} + w\frac{dC}{dz} = D\left(\frac{d^2C}{dx^2} + \frac{d^2C}{dy^2} + \frac{d^2C}{dz^2}\right), \quad (1)$$

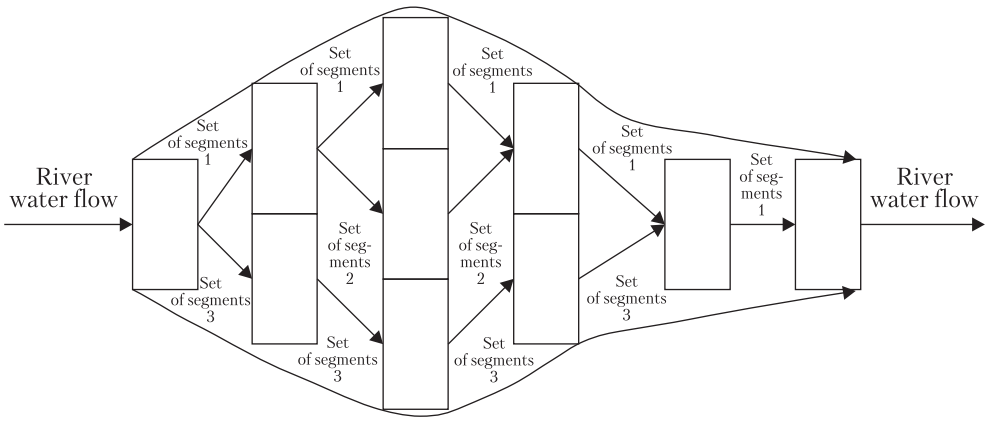

Fig. 3. Grid model that represents the riverbed

where *x, y,* and *z* are the stream directions, *u*, *v,* and *w* are the constant velocities in directions *x, y,* and *z*, respectively, *C* is the average concentration, D is the longitudinal diffusion coefficient, *t* is time.

At the same time, the gradient is described in three directions, each having its own diffusion coefficient that is derived from specialized formulas, such as those described by Kashefipour and Falconer [8].

Other models for water quality assessment in a river work in a similar way. The described principle has a sufficiently high level of accuracy in the case of modeling the processes of migration of substances, i.e. the channel is considered in sufficient detail. The level of detail increases due to the description of the channel itself. At the same time, this approach may be disadvantageous if the hydrological characteristics of the riverbed are unknown, but information about released toxic substances effect is necessary.

Given the tasks and shortcomings of the construction of a gradient of values, the most successful principle underlying this mathematical method is the results interpolation method [9]. The general formula for the interpolation is as follows:

$$y = \frac{1}{n}\sum_{i=1}^{n} y_i. \qquad (2)$$

The purpose of the modification is to find a value in the middle between two points. The formula for finding the concentration peak (*Cp*) for

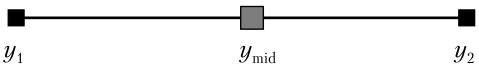

Fig. 4. The searched parameter for y_{mid} between y_1 and y_2

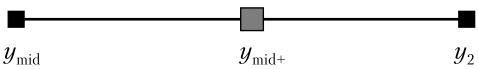

Fig. 5.The searched parameter for y_{mid+} between y_{mid} and y_2

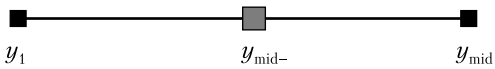

Fig. 6. The searched parameter for $y_{mid–}$ between y_1 and y_{mid}

an arbitrary point in the middle between two points has the following form:

$$Cp_{mid} = \frac{Cp_1 + Cp_2}{2}, \qquad (3)$$

where Cp_1 is the concentration peak in point 1, Cp_2 is the concentration peak in point 2, Cp_{mid} is the concentration peak in the middle point.

Similarly, we may determine the time when the concentration of the substance reaches its peak. The formula has the following form:

$$Tp_{mid} = \frac{Tp_1 + Tp_2}{2}, \qquad (4)$$

where Tp_1 is the time when the concentration reaches its peak in point 1, Tp_2 is the time when the

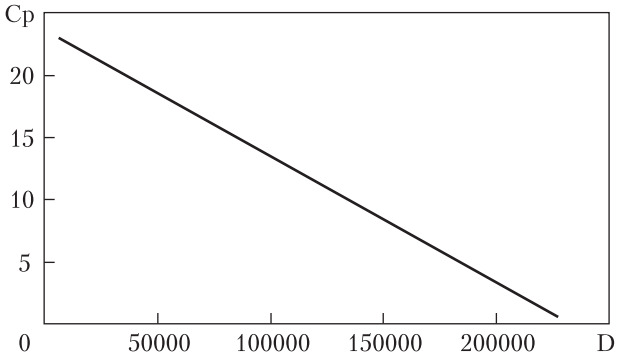

Fig. 7. Relationship between peak concentration on distance, without the use of the method

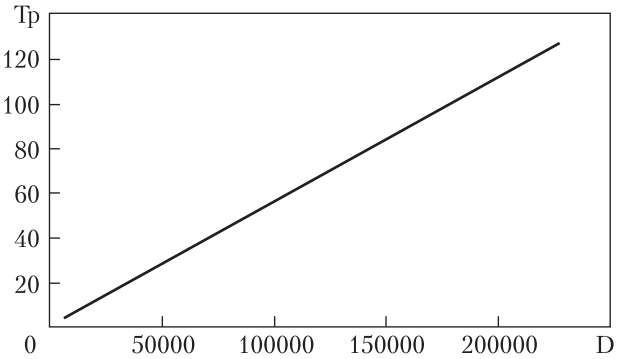

Fig. 8. Relationship between the time when the concentration reaches its peak and the distance, without using the method

Table 1. **Input Values of Point 1 and Point 2**

Point No.	Distance from the initial point	Time when the concentration reaches its peak	Peak concentration
1	6950.318	4.28	22.97
2	226580.5	126.07	0.64

Table 2. **Results Calculated at the First Level of recursion with the Use of the Method**

Point No.	Distance from the initial point	Time when the concentration reaches its peak	Peak concentration
1	6950.318196	4.28	22.97
	61857.8639	34.7275	17.3875
	116765.4096	65.175	11.805
	171672.9553	95.6225	6.2225
2	226580.501	126.07	0.64

concentration reaches its peak in point 2, Tp_{mid} is the time when the concentration reaches its peak in the middle point.

The place is determined by the logic of deviation from the first point in a similar way. However, it is necessary to determine the distance from point 1 to point 2. The calculation formula is as follows:

$$D = D_2 - D_1, \quad (5)$$

where D_1 is the distance from the first point to point 1, D_2 is the distance from the first point to point 2, D is the distance from point 1 to point 2. The middle between points 1 and 2 is calculated by the formula:

$$D_{mid} = \frac{D}{2}, \quad (6)$$

where D_{mid} is the distance to the middle. The distance from point 1 to the searched point is determined by the formula:

$$D_S = D_1 + D_{mid}, \quad (7)$$

where D_1 is the distance from the first point to point 1, D_{mid} is the distance to the middle point, D_S is the distance to the searched point between points D_1 and D_2.

The described approach allows determining the middle point between points D_1 and D_2. Thus, we have a 1.5-time larger array that reflects the dynamics. The array size may be not enough, especially when it is necessary to determine the concentration of the substance located in the riverbed over long distances.

In this case, it is necessary to use the recursion algorithm that allows determining the middle point between point 1 and the middle point between point 1 and 2.

In the same way, we may find the middle point between D_1 and D_2, as well as between D_S and D_2. The formulas representing this principle have the following form:

$$y_{mid-} = \frac{y_1 + y_{mid}}{2}, \quad (8)$$

where y_1 is the searched parameter in point 1, y_{mid} is the searched parameter in the middle point between y_1 and y_2, y_{mid-} is the searched parameter in the middle point between y_1 and y_{mid}.

Similarly:

$$y_{mid+} = \frac{y_{mid} + y_2}{2}, \quad (9)$$

where y_2 is the substance concentration in point 2, y_{mid+} is the searched parameter in the middle point between y_{mid} and y_2. Schematically, the interpolation logic is shown in Figs. 4, 5, and 6.

Thus, using this logic, it is possible to obtain a sufficient array of values to reflect substance concentration distribution dynamics in the riverbed. The distance difference can be a condition to complete recursion. A 200 m distance difference between neighboring points may mean that the recursion corresponding level calculations are the last ones.

The correctness of the method can be proven graphically. The main criterion is the graphs similarity displaying the values without interpolation and the output array formed as an interpolation result.

In Table 1, there are given values in 2 points, the peak concentration and the time when the concentration reaches its peak.

The graphs for the given points are shown in Figs. 7 and 8.

Using the method at the first regression level is sufficient to prove the method correctness and to obtain the results array shown in Table 2.

The mentioned results are shown in Figs. 9 and 10.

Having analyzed the graphs in Figs. 7—8 and 9—10, we find that they are identical. This means that the method does not create errors, and therefore the correctness of this method can be considered proven.

Therefore, using the method together with a series of Harvey Jobson's empirical hydrological equations, as part of results intelligent processing makes it possible to predict the pollution level between riverbed specific points by mathematical modeling, with the help of interpolation and recursion.

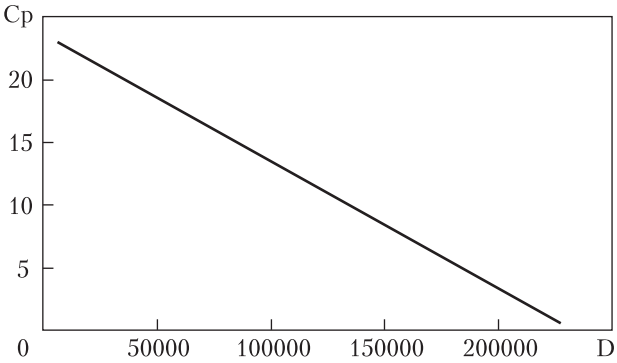

Fig. 9. Relationship between the peak concentration and the distance, with the use of the method

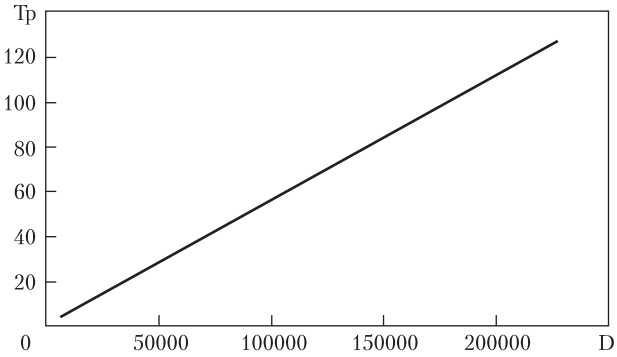

Fig. 10. Relationship between the time it takes for the concentration to reach its peak and the distance using the method

This method involves creating the data array that describes pollution spread dynamics in time and space. The method gives results for the peak concentration, the time when the concentration reaches its peak, and the distance between points.

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ПРОГНОЗУВАННЯ РІВНЯ ЗАБРУДНЕННЯ МІЖ ТОЧКАМИ МЕТОДОМ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ ЗА ДОПОМОГОЮ ІНТЕРПОЛЯЦІЇ ТА РЕКУРСІЇ

Вступ. Існують різні методи математичного моделювання оцінки наслідків забруднення русла річок. Одним з таких є серія емпіричних гідрологічних рівнянь Харві Джобсона, що дозволяє здійснювати розрахунки між точкою забруднення та необхідною точкою обчислення. Обчислення результатів між двома необхідними точками, що знаходяться послідовно за руслом річки досягається шляхом проведення двох обчислень для цих точок та співставлення результатів, що дає можливість прогнозування динаміки поширення забруднення русла річки в часі та просторі.

Проблематика. Прогнозування результатів за двома точками, вірогідно, буде непереконливим оскільки результатом є масив недостатньої величини. Таким чином, виникає потреба розширення результатів масиву даних для збільшення уявлення про точність розрахунків динаміки поширення забруднення.

Мета. Розробка методу, що визначає рівень забруднення між визначеними точками вимірювання та не потребує додаткових даних. Результуючими характеристиками для масиву є пік концентрації та час, через який концентрація набуде свого піку в певній точці водойми.

Матеріали й методи. Основою для побудови математичного методу є метод інтерполяції результатів в поєднанні з методом рекурсії, в результаті чого створюється масив значень для проміжних точок за характеристиками піку концентрації та часу, через який концентрація набуде свого піку.

Результати. Розроблено та апробовано метод розрахунку рівня забруднення в двох послідовних точках шляхом проведення серії симуляцій. Валідність методу доведено графічно, де головним критерієм є схожість графіків.

Висновки. Розроблений метод при використанні разом із серією емпіричних гідрологічних рівнянь Харві Джобсона як частина інтелектуальної обробки результату дає можливість прогнозувати рівень забруднення між конкретними точками русла річки за допомогою методів інтерполяції та рекурсії, використовуючи математичне моделювання, що підвищує прогнозування результатів наслідків надзвичайних ситуацій.

Ключові слова: оцінка якості водойм, русло річки, прогнозування наслідків забруднення водойм, математичне моделювання, інтерполяція, рекурсія.