THEORY OF INFORMATION TECHNOLOGIES AND SYSTEMS CONSTRUCTION

ТЕОРІЯ ПОБУДОВИ ІНФОРМАЦІЙНИХ ТЕХНОЛОГІЙ ТА СИСТЕМ

ttps://doi.org/10.15407/intechsys.2026.04.007
UDC 004.92

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USING THE FOURIER POLYNOMIAL WITH FRACTIONAL TERMS FOR IMPLICIT SURFACE MODELING

Implicit surfaces are widely used in computer modeling due to their ability to model complex objects with relatively little data. Also, in contrast with explicit ways of modelling, such as with triangle meshes and boundary representation of parametric surfaces, the implicit surfaces are guaranteed to keep continuity and smoothness under the operations of deformation or replication.

The explicit modeling, however, such as triangle mesh or boundary representation of parametric surfaces, is the de facto current standard of computer modeling. The problem of conversion from the explicit representation to an implicit one is therefore a cutting-edge engineering problem. New approaches and specific methods are published every year, including those that are based on machine learning or artificial intelligence in general.

Cite: Ausheva N.M., Sydorenko Iu.V., Kaleniuk O.S., Saiapin S.P. Using the Fourier Polynomial with Fractional Terms for Implicit Surface Modeling. Information Technologies and Systems. 4 (10). 2026. 7—15. https://doi.org/10.15407/intechsys.2026.04.007

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In this work, we explore the limits of a triangle mesh to implicit surface conversion based on Fourier polynomial approximation, although the polynomial we propose contains fractional terms. The prospects of using a polynomial with solely integer terms are limited by the Fourier polynomial's tendency to overfit. We show that adding fractional terms to the integer ones in a single function allows for reaching the modeling quality that could be satisfactory for some practical scenarios, while keeping the total amount of polynomial coefficients relatively small too.

In theory, using the Fourier polynomial with fractional terms could be used not only for smooth surface modeling but for smooth connection of different triply periodic surfaces along a selected plane due to the planned loss of periodicity along the normal axis. This, however, requires studying the question of specific per-axis polynomial configuration, which extends beyond the scope of this particular paper.

Keywords: *modeling, geometric modeling, Fourier polynomial, approximation, implicit surfaces, smooth surfaces.*

Introduction

One well-known method of turning an explicit geometric model, such as a triangle mesh, into an implicit surface is meta-balls [1] or similar meta-objects based on triangles [2]. An alternative, but also well-established, approach would be building an implicit surface as the convolution surface of a Gaussian kernel [3]. This approach can be extended to use other kernels as well.

A similar approach would be building implicit surfaces based on radial basis functions [4]. The target implicit surface is then an isosurface of a spatial interpolating function. To localize the influence of each radial basis function, and save computing time, respectively, as otherwise all basis functions would have been computed for every queried point in space, in practice, the compactly supported radial functions are often used instead of globally supported functions [5, 6].

Contemporary approaches usually rely on neural networks to represent a signed distance function of an implicit surface, or the occupancy probability for the target shape [7, 8]. The downsides of machine learning methods are high consumption of computer memory and processing time, both for training the neural network and computing the resulting model. This effect is more pronounced on highly detailed models, so to mitigate it, the neural networks are often enhanced by employing Fourier basis functions for greater detail [9, 10, 11].

The Fourier polynomial by itself is periodic and could be used for triply periodic surfaces modeling purely by itself, without neural networks or any other means of machine learning. A simple least squares approximation combined with an interpolating polynomial retrieved by solving a linear system often grants a viable result on its own [12].

Using the Fourier approximation alone to model non-periodic surfaces, however, is questionable not only due ot its periodicity, after all, every closed surface can be represented as a periodic one with its own bounding box dimensions being the per-axis periods, but mostly due to the Fourier polynomial tendency to overfit.

Problem Statement

Let there be a model of a 3D body presented by the triangle mesh of its surface, *TM*. The triangle mesh consists of n vertices $v_i = (x_i, y_i, z_i)$, $i = 1\ldots n$, and m trinagles $t_j = [v_{j1}, v_{j2}, v_{j3}]$, $j = 1...m$. The triangle mesh is closed, meaning that there is a clear distinction between the points in space lying inside the modeled body and the points outside. Topological inconsistencies that don't affect this distinction, such as non-manifold edges, duplicate triangles, or degenerate triangles, are allowed.

The function that defines a signed distance to *TM* for the value of every point in space is called a signed distance function: $SDF_{TM}(x, y, z)$. We expect the function to signify inner body points with a negative sign and outer ones with a positive. Of course, all points of the *TM* triangles have an SDF_{TM} value of 0.

Our goal is to find a function $F(x, y, z)$ that models the $SDF_{TM}(x, y, z)$ in the axis-oriented bounding box of a modeled body, specifically.

To oversee the overfitting effect, the quality of modeling should be assessed by the set of points and corresponding SDF_{TM} values independent from the ones used to find the $F(x, y, z)$.

Purpose of the Paper

This paper is meant to demonstrate how adding terms with fractional variable multipliers allows both to move away from periodicity and improve the modeling quality. With these terms, modeling smooth surfaces represented by triangle meshes with approximating Fourier polynomials becomes possible up to some level of accuracy, and this could be enough for some practical applications. The model then remains compact memory-wise and relatively simple to compute.

Results

The univariate Fourier polynomial is a partial sum of the Fourier series. The exact formula depends on whether the degree of the polynomial p is even (1) or odd (2):

$$F_e(x) = a_0 + a_1\cos(x) + a_2\sin(x) + \\ + a_3\cos(2x) + a_4\sin(2x) + \ldots + a_{p-1}\cos\left(\frac{p}{2}x\right) + a_p\sin\left(\frac{p}{2}x\right) \quad (1)$$

or

$$F_0(x) = a_0 + a_1\cos(x) + a_2\sin(x) + \\ + a_3\cos(2x) + a_4\sin(2x) + \ldots + a_p\cos\left(\frac{p+1}{2}x\right). \quad (2)$$

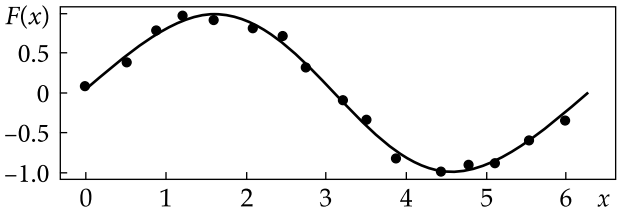


Fig. 1. A Fourier polynomial of degree 5 approximating the sine function represented by 16 points with random noise

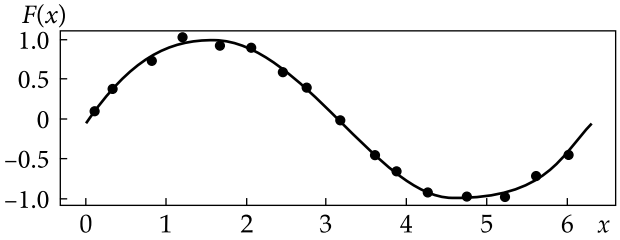


Fig. 2. A Fourier polynomial of degree 10 approximating the sine function represented by 16 points with random noise

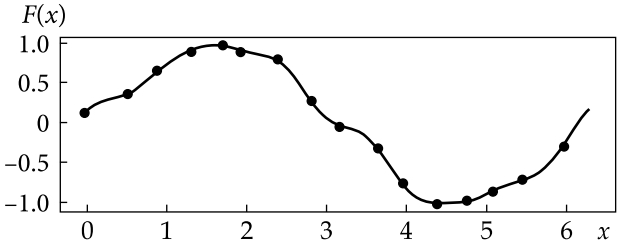


Fig. 3. A Fourier polynomial of degree 15 approximating the sine function represented by 16 points with random noise

This polynomial is periodic with period [0..2π]. It is also a single interpolating polynomial for a set of $p + 1$ points. And while the Fourier series converges to any algebraic function, the Fourier polynomial shows a tendency to overfit when using it to approximate a function by its values with noise (Fig. 1–3).

We can build a similar polynomial based on Fourier terms where the degree of the term becomes not a multiplier but a divisor of an argument in the corresponding trigonometric function (3), (4). Let's call this polynomial a fractional Fourier polynomial (FFP).

$$F_e(x) = a_0 + a_1\cos(x) + a_2\sin(x) + a_3\cos(x/2) + \\ + a_4\sin(x/2) + \ldots + a_{p-1}\cos\left(\frac{2x}{p}\right) + a_p\sin\left(\frac{2x}{p}\right). \quad (3)$$

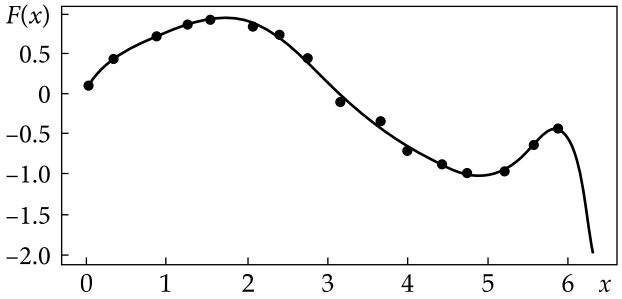


Fig. 4. An FFP of degree 15 approximating the sine function represented by 16 points with random noise

$$F_0(x) = a_0 + a_1\cos(x) + a_2\sin(x) + \\ + a_3\cos(x/2) + a_4\sin(x/2) + \ldots + a_p\cos\left(\frac{2x}{p+1}\right). \quad (4)$$

This polynomial shows less tendency to overfit (Fig. 4) but also loses the $[0..2\pi]$ periodicity. Of course, for the purpose of modeling a non-specific smooth surface given by a triangular mesh, periodicity isn't a desired property.

Experiments show, however, that this polynomial has less descriptive potential, especially regarding small model features. To mitigate this issue, we can build a polynomial that includes both regular Fourier terms (with multipliers) and fractional ones (with divisors) (5). Let's call this polynomial a Fourier polynomial with fractional terms (FPFT):

$$F(x) = a_0 + a_1\cos(x) + a_2\sin(x) + \ldots + \\ + a_{p-3}\cos\left(\frac{p+1}{4}x\right) + a_{p-2}\sin\left(\frac{p+1}{4}x\right) + a_{p-1}\cos\left(\frac{4x}{p+1}\right) + a_p\sin\left(\frac{4x}{p+1}\right). \quad (5)$$

To evaluate the descriptive potential of this polynomial, we conducted a series of computational experiments. As a prototype for modeling, we chose a triangle mesh representing a "Stanford bunny" (Fig. 5).

To model this particular mesh with FPFT implicit surface, we first rescaled the model so it would fit the $[-7/8\pi; 7/8\pi]\times[-7/8\pi; 7/8\pi]\times$ $\times[-7/8\pi; 7/8\pi]$ cube. The coefficient 7/8 was picked to retain a 1/4 margin in the $[-\pi;\pi]\times$ $\times[-\pi;\pi]\times[-\pi;\pi]$ cube, where otherwise we would have faced distortions caused by 2π-periodic terms of the FPFT.

Then, in the $[-\pi;\pi]\times[-\pi;\pi]\times[-\pi;\pi]$ cube, we generated a regular grid of points with own dimensions: 32 × 32 × 64. The last dimension is intentionally disproportionate so we could

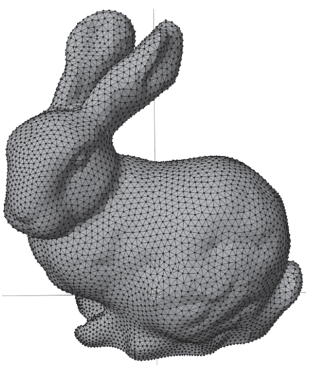


Fig. 5. A finely detailed triangle mesh of the "Stanford bunny"

easily divide the grid into two independent sets of points: one for approximation and the other for validation. Each even point of the grid, along with the signed distance value to the model from that point, was included in the approximation set. Each even point, along with its signed distance value, was included in the validation set.

Just like that, each even vertex of the source triangle mesh, along with value 0, joined the approximation set, and each even vertex, also with a signed distance value of 0, joined the validation set.

In our experiment, we tried all three polynomials: Fourier polynomial (FP), fractional Fourier polynomial (FFP) and Fourier polynomial with fractional terms (FPFT), each generalized to the tree argument function by using a full Cartesian product of its terms. Specifically, we tested polynomials (1), (3) and (5), all with odd degrees, specifically: 7, 11, 15, 19. Degree 7 was picked as a minimal one since this is the first even degree, for which FPFT becomes different from a regular Fourier polynomial. By the same logic, the degree increment of 4 is the minimal increment that keeps all three polynomials different.

For each polynomial, we built an implicit surface by fitting the polynomial into the approximation point set by using conventional least square approximation. For the set of approximating points (x_i, y_i, z_i) along with signed distance to the source mesh values v_i, $i = 1..N$, the coefficients of polynomials F were obtained by the following criterion (6):

$$(F(x_i, y_i, z_i) - v_i)^2 \to \min. \tag{6}$$

Then, to validate the modeling accuracy, for each found polynomial, the root mean square error was computed in all the points of both approximation (RMSEa) and validation (RMSEv) sets. The collected data is presented in Table 1.

The same RMSEa data is also shown in Figure 6.

Surprisingly, the data shows that the Fourier polynomial of degree 19 doesn't show signs of overfitting, although its error still remains larger than for every other polynomial. The FFP doesn't show a consistent picture regarding the modeling accuracy, and the FPFT shows the signs of overfitting in degree 19, while still retaining the least error in lesser degrees.

Table 1. **Root mean square error values for Fourier polynomials**

Degree	FP		FFP		FPFT	
	RMSEa	RMSEv	RMSEa	RMSEv	RMSEa	RMSEv
7	6.10E-05	6.93E-05	2.50E-06	2.60E-06	1.78E-06	1.82E-06
11	3.73E-05	4.55E-05	3.78E-06	4.20E-06	5.08E-07	5.63E-07
15	2.43E-05	3.24E-05	7.44E-06	8.77E-06	9.63E-07	2.21E-06
19	1.62E-05	2.40E-05	3.26E-06	4.05E-06	3.71E-06	2.07E-05

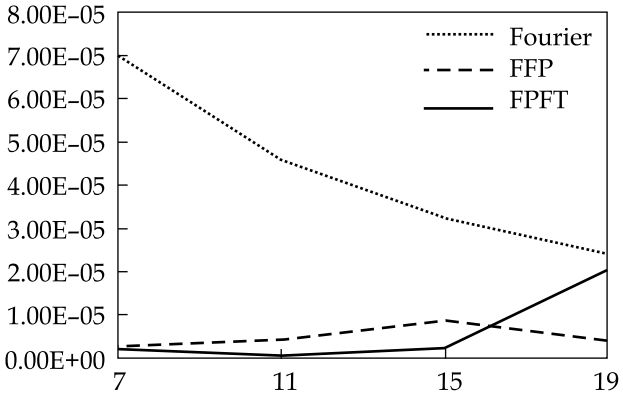


Fig. 6. RMSEa for different types of Fourier polynomials

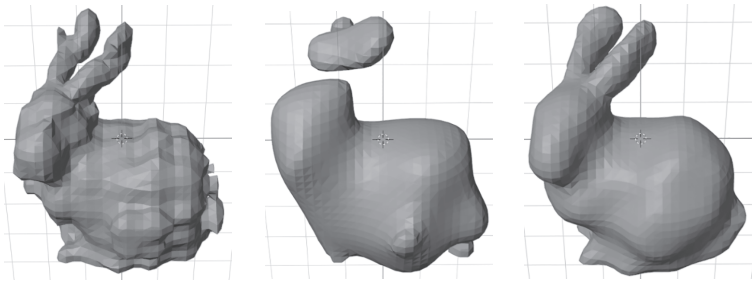


Fig. 7. Visual comparison of Fourier polynomials' modeling quality

For the visual comparison, we provide three reconstructions of implicit surfaces obtained with different polynomials of degree 15.

In Figure 7, A is the product of regular Fourier polynomial approximation, B is FFP, and C is FPFT. It's apparent that the Fourier polynomial with fractional terms is the most applicable for surface modeling among the three.

Conclusions

The quality of implicit surface modeling with a trigonometric polynomial could be enhanced drastically by introducing fractional terms into the approximating polynomial. The improvement, however, is still limited by the overfitting effect, especially on the models with highly detailed surfaces, or on non-smooth surfaces where the regularity of the underlying function isn't wanted.

The interesting practical problem to solve would be to find a way to configure the polynomial for the target model to approximate automatically. In this work, we decided to limit the polynomials to 7, 11, 15 and 19 degrees specifically so the number of integer and fractional terms would be even. But is this necessary or even desired? Perhaps this particular surface could have been better modeled with a different polynomial configuration. This question requires a separate investigation.

DECLARATION

Author contributions: *Ausheva N.M. – Supervision, Project administration; Sydorenko Yu.V. – Methodology, Writing – Review & Editing; Kaleniuk O.S. – Conceptualization, Software, Writing – Original Draft; Saiapin S.P. – Validation, Writing – Review & Editing.*

AI and Tools Usage: *The authors declare that generative AI has not been used for the creation of this paper in any capacity.*

Conflict of interest: *The authors declare that there is no conflict of interest.*

Grant or Financial Support: *The research has been conducted without a grant or financial support.*

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Received 29.06.2026
Accepted 30.07.2026
Published 23.10.2026

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ЗАСТОСУВАННЯ ПОЛІНОМА ФУР'Є ІЗ ДРОБНИМИ ЧЛЕНАМИ ДЛЯ МОДЕЛЮВАННЯ ГЕОМЕТРИЧНИХ ОБ'ЄКТІВ ІМПЛІЦИТНИМИ ПОВЕРХНЯМИ

Імпліцитні поверхні широко застосовуються у комп'ютерному моделюванні завдяки їх властивості моделювати складні об'єкти відносно малою кількістю даних. Також важливо, що на відміну від моделей із експліцитним представленням елементів, імпліцитні поверхні не втрачають неперервності, гладкості чи регулярності при операціях деформації або реплікації. Разом із тим, експліцитні моделі, наприклад на основі трикутникової сітки, або на основі параметричного граничного представлення є золотим стандартом комп'ютерного моделювання. Для роботи з такими моделями існує безліч інструментів, бібліотек програмного коду, загальновідомих алгоритмів тощо. Питання конвертації із експліцитного представлення геометричної моделі в імпліцитне лишається питанням передового краю науки, щороку публікуються нові сучасні методики, які ґрунтуються зокрема і на досягненнях у сфері машинного навчання і штучного інтелекту.

Мета даної роботи — дослідити межі моделювання поверхні, початково заданої трикутниковою сіткою, апроксимаційним модифікованим поліномом Фур'є, який містить в собі дробні члени. Поліном Фур'є задає функцію знакової відстані, нульова ізолінія такої функції має описувати вхідну поверхню, початково задану трикутниковою сіткою, якнаймога точніше.

Методи. Основним методом дослідження є порівняльний аналіз деяких модифікацій полінома Фур'є, здійснене за допомогою обчислювальних експериментів.

Результати. Перспективи застосування полінома Фур'є без додаткових модифікацій є вельми обмеженими завдяки схильності апроксимаційного полінома до оверфітінгу, але ми показуємо, що додавання дробних членів до полінома дозволяє досягнути задовільної для певних задач точності моделювання при використанні відносно невеликої кількості коефіцієнтів полінома.

Висновки. Застосування полінома Фур'є із дробними членами дозволяє не просто моделювати замкнені геометричні об'єкти з гладкими поверхнями, а і, в перспективі, будувати поверхні спряження для інших три-періодичних поверхонь при втраті періодичності по одній з осей.

Ключові слова: *моделювання, геометричне поделювання, поліном Фур'є, апроксимація, неявне завдання поверхонь, гладкі поверхні.*