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A COMPARATIVE ECONOMETRIC STUDY OF ARIMA-GARCH MODELS IN MODELING SAUDI ARAMCO STOCK VOLATILITY

This study models Saudi Aramco stock volatility (2022—2025) using a hybrid ARIMA-CS-GARCH framework. Among over 750 specifications, ARIMA(2,1,2)-CS-GARCH(0,1) best captures long- and short-term volatility, outperforming conventional ARIMA in out-of-sample forecasts, with implications for investors and risk managers.

Keywords: stock volatility; Saudi Aramco; ARIMA-GARCH models; financial markets.

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Financial market volatility represents a major challenge for investors, policymakers, and risk managers, as it directly affects investment decisions, portfolio allocation, and financial stability. Financial time series are characterized by volatility clustering, conditional heteroskedasticity, and asymmetric responses to shocks, which limit the ability of traditional linear models such as ARIMA to capture market dynamics.

To address these limitations, recent studies have increasingly adopted hybrid ARIMA-GARCH and CS-GARCH frameworks that jointly model linear dependencies in the conditional mean and nonlinear dynamics in conditional variance. These models have demonstrated superior performance across various asset classes, including equities, oil prices, and cryptocurrencies, due to their ability to capture both short-term fluctuations and long-term volatility persistence.

Despite these advances, the application of hybrid ARIMA-CS-GARCH models to major Saudi stocks remains limited. In particular, few studies have examined the decomposition of volatility into permanent and transitory components or evaluated out-of-sample forecasting performance using recent high-frequency data. Given the strategic importance of Saudi Aramco in both domestic and global energy markets, a comprehensive volatility modeling framework is needed.

Accordingly, this study analyzes the daily adjusted closing prices of Saudi Aramco over the period from January 2, 2022 to May 8, 2025. It aims to assess the suitability and forecasting performance of the ARIMA(2,1,2)-CS-GARCH(0,1) model in capturing conditional volatility dynamics. Specifically, the study addresses the following research questions:

- How effectively does the ARIMA (2,1,2)-CS-GARCH(0,1) model capture volatility dynamics in Saudi Aramco stock prices?
- Does the hybrid model provide more accurate out-of-sample forecasts than the conventional ARIMA model?
- How can the model's results support investment and risk management decisions in a volatile market environment?

Recent empirical and methodological literature provides evidence that linear ARIMA models are limited in their ability to capture stylized facts of financial time series, particularly volatility clustering and conditional heteroskedasticity. This limitation has motivated the widespread adoption of hybrid ARIMA-GARCH frameworks, which combine mean and variance modeling to better represent financial dynamics.

A first strand of literature demonstrates the superiority of hybrid ARIMA-GARCH models over standalone linear specifications. C. Dretsaki (2018) shows that combining ARIMA with GARCH significantly improves the modeling of oil price returns, while K. Sun (2017) confirms similar improvements for U.S. index returns, as evidenced by information criteria and out-of-sample evaluation. N. Vo & R. Slepaczuk (2022) find that hybrid ARIMA-GARCH models outperform ARIMA in both forecasting accuracy and trading performance under rolling-window analysis. A second strand emphasizes the importance of asymmetric volatility modeling and distributional assumptions. A. Hossain et al. (2015) and O. Adegboyo & K. Sarwar (2025) demonstrate that asymmetric GARCH specifications, particularly EGARCH and GJR-GARCH under heavy-tailed distributions, better capture the differential

impact of positive and negative shocks. These findings are consistent across both developed and emerging markets. A third strand highlights volatility persistence and long-memory behavior. B. Abu Zarour & C. Siriopoulos (2008) show that CS-GARCH models are particularly suitable for markets characterized by persistent shocks, while T. Liu & Y. Shi (2022) further demonstrate that component-based and distribution-sensitive GARCH models improve forecasting accuracy. Similarly, J. Contreras-Reyes et al. (2024) and S. Benbachir (2025) confirm the presence of long-range dependence in financial volatility, supporting the use of fractional and component-based models.

In addition, recent studies across different markets reinforce the robustness of hybrid and asymmetric models. Evidence from African markets (Wanjuki et al., 2024; Karugano et al., 2023), Asian markets (Wang et al., 2022; Xiang, 2022), and BRICS economies (Singh et al., 2024) consistently shows that advanced GARCH-type models outperform symmetric alternatives, particularly in volatile and structurally unstable environments. Similar conclusions are reached in cryptocurrency and digital asset markets (Sözen, 2025; Queiroz, David, 2023; Silva, Maciel, 2025), where model performance depends strongly on asset-specific characteristics and market conditions.

Despite this extensive literature, several important gaps remain. Empirical applications of hybrid ARIMA-CS-GARCH models to Saudi Aramco stock remain scarce, despite its global significance. In addition, most studies rely on standard GARCH specifications and do not consider volatility decomposition into long- and short-term components. Furthermore, existing evidence on the Saudi market is often based on outdated datasets and lacks rigorous out-of-sample forecasting using recent high-frequency data.

So, the **purpose of the article** is to address these gaps by applying a hybrid ARIMA-CS-GARCH model to decompose the long- and short-term volatility of Saudi Aramco stock and to evaluate its out-of-sample forecasting accuracy using recent daily data.

Research methods. To address these gaps, this study applies a hybrid ARIM(2,1,2)-CS-GARCH(0,1) framework to Saudi Aramco's daily stock prices over the period 2022–2025. It further evaluates multiple error distributions, including Normal, Student-t, and GED, as well as their skewed variants. This approach provides a more accurate representation of volatility dynamics and improves forecasting performance.

DESCRIPTIVE AND DYNAMIC ANALYSIS OF TIME SERIES OF THE ADJUSTED CLOSING PRICE OF SAUDI ARAMCO

The adjusted closing price of Saudi Aramco (SAOC) represents the final stock price after accounting for dividends and stock splits, reflecting the share's true market value. Analyzing this price helps understand overall price behavior, volatility levels, and potential trends, while identifying dynamic features such as volatility clustering and random walk behavior. This analysis guides the selection of appropriate econo-

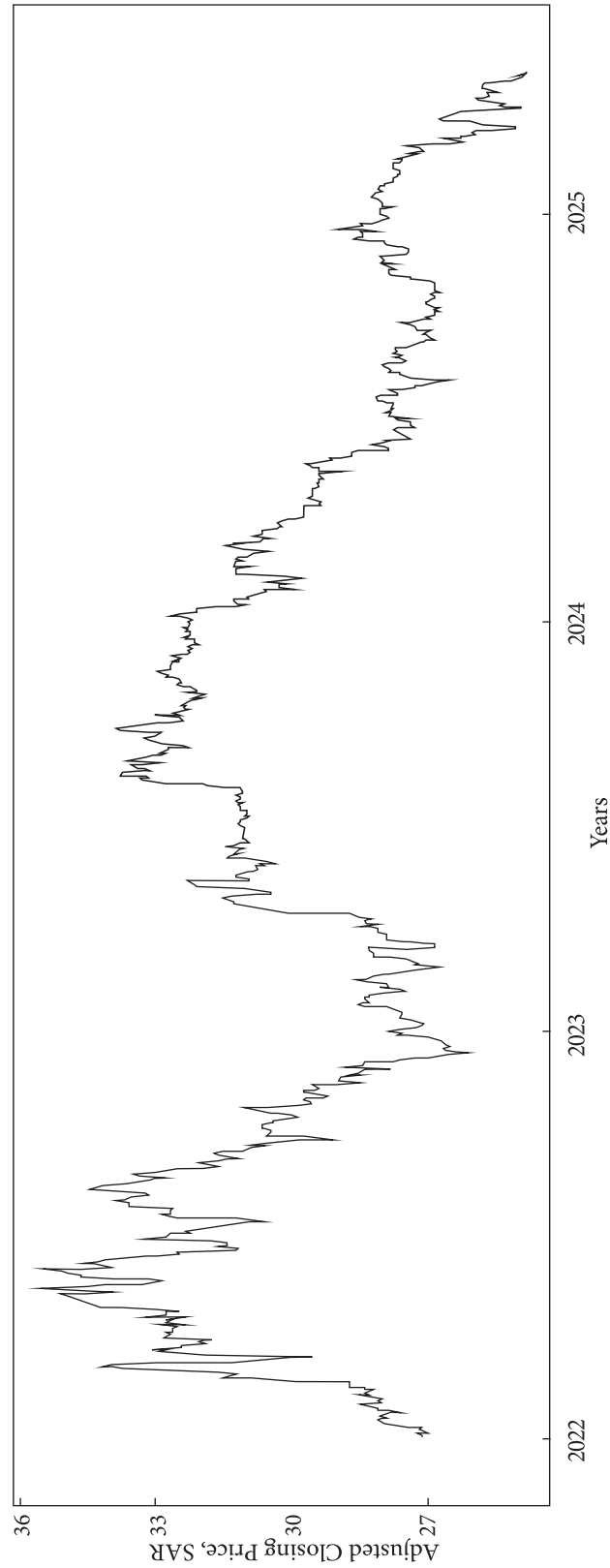


Fig. 1. Daily Time Series of the Adjusted Closing Price of Saudi Aramco
Source: author's elaboration based on: Yahoo Finance. URL: <https://finance.yahoo.com> (accessed: 15.05.2025).

metric models, such as ARIMA for linear patterns and GARCH for conditional variance dynamics (Dritsaki, 2018).

Fig. 1 clearly demonstrates that the time series under study exhibits significant dynamism and volatility. Furthermore, graphical observations reveal a random walk pattern, where both the mean and the variance appear to be time-dependent, which initially indicates the non-stationarity of the time series of the adjusted closing price of Saudi Aramco shares.

EMPIRICAL METHODOLOGY

This study adopts a time-series econometric framework to jointly model the conditional mean and conditional variance dynamics of the analyzed series. Specifically, linear dependencies are captured using ARIMA models, while volatility clustering and heteroskedasticity are modeled through various GARCH-type specifications.

ARIMA Models. The conditional mean of the series is modeled using the classical Autoregressive Integrated Moving Average (ARIMA) framework introduced by Box and Jenkins. An ARIMA (p, d, q) process combines autoregressive terms, differencing operators to ensure stationarity, and moving average components, and can be expressed as (Box et al., 2016, p. 76, 94).

$$\Phi(L)\nabla^d Y_t = c + \Theta(L)\varepsilon_t, \quad (1)$$

$$\Phi(L) = (1 - \varphi_1 L - \varphi_2 L^2 - \dots - \varphi_p L^p), \quad (1.1)$$

where

$$\Theta(L) = (1 - \theta_1 L - \theta_2 L^2 - \dots - \theta_q L^q). \quad (1.2)$$

Its mathematical specification can be written as follows (Wang, 2024, p. 2):

$$Y_t^{(d)} = c + \sum_{i=1}^p \varphi_i Y_{t-i}^{(d)} + \varepsilon_t + \sum_{j=1}^q \theta_j \varepsilon_{t-j}, \quad (2)$$

where p and q denote the orders of the autoregressive and moving average components, respectively, d represents the order of integration, c is a constant term, and ε_t is a white-noise error term. Model stationarity and invertibility conditions are verified following standard criteria.

ARCH Models. Following R. Engle & G. Lee (1982), the ARCH(q) model specifies the variance as a function of past squared innovations (Tsay, 2010, p. 115–116). According to the following general mathematical specification:

$$\varepsilon_t = \sigma_t \zeta_t, \quad (3)$$

where $\{\zeta_t\}$ denotes a sequence of independent and identically distributed random variables with zero mean and unit variance.

As for the conditional variance, it can be specified as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_s \varepsilon_{t-s}^2, \quad (3.1)$$

with $\alpha_0 \succ 0$, $\alpha_i \geq 0$ for $i = 1, \dots, s-1$, and $\alpha_s \succ 0$. These parameter restrictions are imposed to ensure that the conditional variance σ_t^2 is always positive. Moreover, the additional condition $\sum_{i=1}^s \alpha_i \prec 1$ guarantees that the series is covariance-stationary, with a finite unconditional variance σ_a^2 .

GARCH Models. The ARCH model captures the evolution of conditional variance over time but often requires a high lag order. To address this, Bollerslev (1986) introduced the GARCH model, which generalizes ARCH and more effectively models volatility dynamics. It can be expressed according to the following general mathematical specification (Box et al., 2016, p. 366):

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^s \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^r \beta_j \sigma_{t-j}^2, \quad (4)$$

with $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, \dots, s-1$, and $\alpha_s > 0$. In addition to $\beta_j \geq 0$ for $j = 1, \dots, r-1$ and $\beta_r > 0$. These parameter restrictions are sufficient to ensure that the conditional variance σ_t^2 remains strictly positive. D. Nelson & C. Cao (1992) showed that these restrictions can be slightly relaxed, allowing some parameters to be negative while the conditional variance remains positive. The additional condition $\sum_{i=1}^m (\alpha_i + \beta_i) < 1$, where $m = \max(s, r)$ with $\alpha_i = 0$ for $i > s$ and $\beta_i = 0$ for $j > r$, which ensures that the unconditional variance σ_a^2 is finite.

Modern extensions of GARCH models. Because traditional GARCH models fall short of describing the asymmetry of volatility and its dynamic behavior over time, nonlinear extensions have emerged that aim to provide a more flexible and realistic representation of financial volatility.

Component GARCH (CS-GARCH). Traditional GARCH models (Tim Bollerslev) effectively capture short-term conditional volatility but assume constant long-term variance. To address this limitation, the component GARCH (CS-GARCH) model proposed by R. Engle & G. Lee (1999, p. 477) decomposes volatility into long-term (permanent) and short-term (transitory) components. Accordingly, the CS-GARCH model is specified as follows

$$\begin{cases} h_t = q_t + s_t, \\ s_t = (\alpha + \beta)s_{t-1} + \alpha(\varepsilon_{t-1}^2 - h_{t-1}), \\ q_t = \omega + \rho q_{t-1} + \varphi(\varepsilon_{t-1}^2 - h_{t-1}), \end{cases} \quad (5)$$

where s_t represents the short-run (transitory) component, while q_t denotes the long-run (trend) component. As shown by the above equations, the volatility innovation $(\varepsilon_{t-1}^2 - h_{t-1})$ acts as the fundamental driving force behind both components simultaneously (Engle, Lee, 1999, p. 478).

The short-run component tends to revert to its mean (zero) at a geometric rate equal to $(\alpha + \beta)$, provided that $0 < (\alpha + \beta) < 1$. The long-run volatility component evolves according to an autoregressive process and converges to a constant level when $0 < \rho < 1$. The long-run component is more persistent than the short-run component, implying that $0 < (\alpha + \beta) < 1$.

EGARCH Model. Proposed by D. Nelson (1991, p. 350—351), the EGARCH (p, q) model specifies the logarithm of the conditional variance as

$$\ln(h_t) = \alpha_t + \sum_{k=1}^{\infty} \beta_k g(z_{t-k}), \quad (6)$$

where

$$g(z_t) \equiv \theta z_t + \gamma(|z_t| - E|z_t|). \quad (7)$$

This formula ensures a permanent positive representation of variances without the need to impose constraints on coefficients, and it also allows for the representation of the asymmetric response to fluctuations.

TGARCH/GJR-GARCH Model. To explicitly model asymmetric responses to negative shocks by incorporating a conditional variable (indicator function) that is activated only when negative shocks occur (Glosten et al., 1993, p. 1787) proposed the TGARCH model:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^s (\alpha_i + \gamma_i I_{t-i}) \varepsilon_{t-i}^2 + \sum_{j=1}^r \beta_j \sigma_{t-j}^2. \quad (8)$$

NGARCH Model. R. Engel & V. Ng (1993, p. 1755) introduced the NGARCH model, which incorporates nonlinearity and asymmetry directly into the variance equation:

$$h_t = \omega + \beta h_{t-1} + \alpha (\varepsilon_{t-1} + \gamma \sqrt{h_{t-1}})^2. \quad (9)$$

The minimum point of the curve occurs at $\varepsilon_{t-1} = (-\gamma) \cdot \sqrt{h_{t-1}}$, and it shifts to the right of the origin, when γ is negative, implying that negative shocks generate higher volatility than positive shocks.

APARCH model. Z. Ding et al. (1993, p. 98) proposed the APARCH model, which allows for asymmetry, nonlinearity, and varying degrees of volatility responses to shocks on magnitude and direction.

$$\varepsilon_t = \sigma_t \zeta_t, \quad (10)$$

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^q \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^p \beta_j \sigma_{t-j}^\delta, \quad (11)$$

where $\alpha_0 > 0$, $\delta_i \geq 0$ with $\alpha_i \geq 0$ for $i = 1, \dots, p$. It also is required that $-1 < \gamma_i < 1$ for $i = 1, \dots, p$, allows the model to capture degree of asymmetry between positive and negative shocks. In addition, $\beta_j \geq 0$ for $j = 1, \dots, q$.

FIGARCH Model. To capture long-memory properties in volatility dynamics, R. Baillie, T. Bollerslev, & H. Mikkelsen (1996, p. 8) introduced the FIGARCH model:

$$[1 - \beta(L)] \sigma_t^2 = \omega + [1 - \beta(L) - \varphi(L)(1-L)^d] \varepsilon_t^2, \quad (12)$$

where $d \in (0,1)$ denotes the fractional integration parameter.

Hybrid ARIMA-GARCH Model. To jointly model linear dynamics in the mean and nonlinear volatility behavior, a hybrid ARIMA-GARCH framework is adopted. The conditional mean is specified as:

$$\begin{cases} Y_t^{(d)} = \mu_t + \varepsilon_t, \end{cases} \quad (13)$$

$$\begin{cases} \mu_t = c + \sum_{i=1}^p \varphi_i Y_{t-i}^{(d)} + \sum_{j=1}^q \theta_j \varepsilon_{t-j}, \end{cases} \quad (14)$$

while the conditional variance follows a SC-GARCH process:

$$\begin{cases} h_t = q_t + s_t, \\ s_t = (\alpha + \beta) s_{t-1} + \alpha (\varepsilon_{t-1}^2 - h_{t-1}), \\ q_t = \omega + \rho q_{t-1} + \varphi (\varepsilon_{t-1}^2 - h_{t-1}). \end{cases} \quad (15)$$

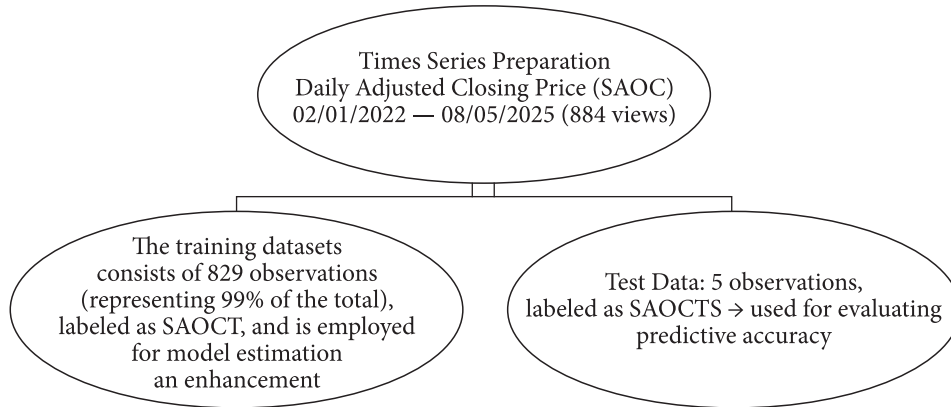


Fig. 2. Steps to Estimate the Model
Source: computed by the authors.

Thus, the ARIMA-CS-GARCH model offers a comprehensive and flexible framework for analyzing the time series, enabling the separation and estimation of both the linear dynamics in the mean equation and the dual short- and long-term fluctuations in the conditional variance.

MODEL SELECTION CRITERIA

Model selection is conducted using standard information criteria:

$$\begin{cases} AIC = -2 \ln(L) + 2k, \\ BIC = -2 \ln(L) + \ln(N)k, \\ HQC = -2 \ln(L) + 2 \ln(\ln(N))k, \end{cases} \quad (16)$$

where L denotes the maximized likelihood function, k — the number of estimated parameters, and N — the sample size. The model that achieves the lowest value among the AIC , SBC , or HQC criteria is considered the best model when making comparisons.

FORECAST ACCURACY MEASURES

Forecasting performance is evaluated using the following loss functions:

$$MSE = \frac{1}{T} \sum_{t=T_1}^T (\sigma_t^2 - \hat{\sigma}_t^2)^2, \quad RMSE = \sqrt{\frac{1}{T} \sum_{t=T_1}^T (\sigma_t^2 - \hat{\sigma}_t^2)^2},$$

$$MAPE = \frac{1}{T} \sum_{t=T_1}^T \left| \left(\frac{\sigma_t^2 - \hat{\sigma}_t^2}{\sigma_t^2} \right)^2 \right| \times 100.$$

Lower values of these measures indicate superior forecasting accuracy.

Fig. 2 illustrates the division of the time series into segments for the analysis. Note that it does not depict the sequential testing procedures, but rather the structuring of the series.

EMPIRICAL FINDINGS AND DISCUSSION

Descriptive statistics. This subsection presents the descriptive statistics of the SAOC series, summarizing its central tendency, dispersion, and distributional properties, as reported in Table 1.

Table 1 shows that the adjusted closing price reached its highest value in 2022 and its lowest in 2025. The series exhibits moderate dispersion around its mean (SD = 2.435), indicating a relatively stable variation. The skewness coefficient is slightly positive, suggesting a right-skewed distribution, while the kurtosis value is below 3, indicating lighter tails than the normal distribution. Overall, these results suggest that the series deviates from normality.

Table 1. Descriptive Statistics of the Time Series (SAOC)

Descriptive Statistics of SAOC Adjusted Closing Price							
Kurtosis	Skewness	Std. Dev	Max	Min	Median	Mean	Observations
1.883	0.144	2.435	35.615	24.840	29.591	29.821	834

Source: computed by the authors.

Table 2. Stationarity Test Results for SAOCT (ADF and PP)

ADF Test				
	(I1) At first difference		(I0) At Level	
	p-value	Test Statistic	p-value	Test Statistic
None	0.01	-18.8	0.545	-0.344
Constant	0.01	-18.8	0.297	-2.080
Constant & Trend	0.01	-18.8	0.121	-3.080
PP Test				
	(I1) At first difference		(I0) At Level	
	p-value	Test Statistic	p-value	Test Statistic
None	0.01	-590	0.550	-0.331
Constant	0.01	-590	0.429	-1.750
Constant & Trend	0.01	-589	0.246	-2.780

Source: prepared by the authors based on outputs from the R.

Table 3. ARIMA(2,1,2) Estimation Results

Variable	Coefficient	Std. error	T-statistic	p-value
AR(1)	1.3064	0.1247	10.4751	0.00000
AR(2)	-0.5801	0.1308	-4.4362	0.00000
MA(1)	-1.1356	0.1425	-7.9674	0.00000
MA(2)	0.3847	0.1495	2.5725	0.00000

Source: prepared by the authors based on outputs from the R.

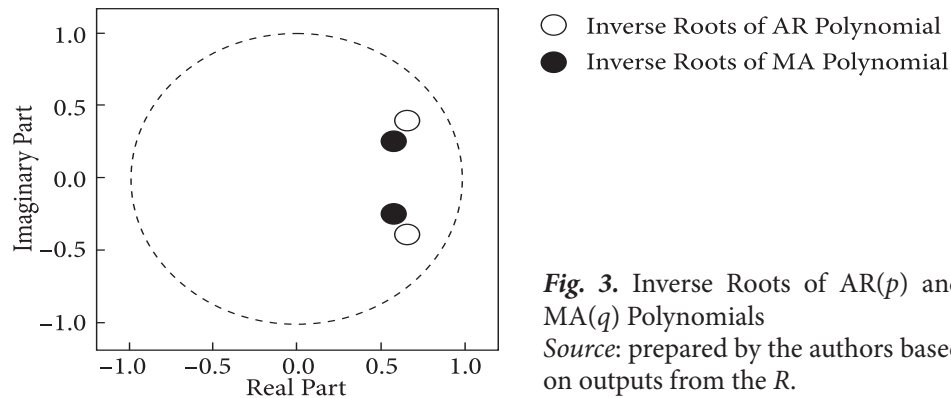


Fig. 3. Inverse Roots of $AR(p)$ and $MA(q)$ Polynomials
Source: prepared by the authors based on outputs from the R.

Stationarity Analysis of the Time Series Data (SAOCT). Stationarity is a prerequisite for ARCH-type models, requiring either a stationary series or an appropriate transformation. To assess this, the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are applied. Table 2 reports the results for the SAOCT training dataset.

Table 2 presents the results of the ADF and PP tests, indicating that the SAOCT series is non-stationary at the 5% significance level, as unit roots are observed across all specifications. Following the application of a logarithmic transformation and first differencing, the series attains stationarity, with no unit roots detected, thereby confirming that it is integrated of order one, $I(1)$.

Model Selection and Parameter Estimation. The SAOCT series was made stationary using a logarithmic transformation and first differences. The $ARIMA(p,d,q)$ model was then fitted, with the orders p and q determined using R 's automatic selection procedure. The $ARIMA(2,1,2)$ model provided the best fit, as indicated by the lowest AIC.

An examination of the results reported in Table 3 above indicates that the estimated parameters obtained from the model estimation process are **highly statistically significant**. Accordingly, the linear equation of the optimal model can be expressed in the following mathematical form:

$$\begin{aligned} \Delta(LSAOCT)_t = & 1.3064 \cdot \Delta(LSAOCT)_{t-1} - \\ & - 0.5801 \cdot \Delta(LSAOCT)_{t-2} - 1.1356 \cdot \varepsilon_{t-1} + 0.3847 \cdot \varepsilon_{t-2} + \zeta_t. \end{aligned} \quad (17)$$

MODEL VALIDATION

The diagnostic stage is also one of the most crucial steps in the modeling process, as it allows for verifying the adequacy and suitability of the initially proposed model in representing the observed data. This is achieved by conducting a set of diagnostic tests, among which the most important are highlighted below:

Stability and Invertibility Analysis. Verifying the conditions of stability and invertibility of the estimated model parameters constitutes a key indicator of the model's efficiency and its suitability for representing the study data. To ensure that the model proposed in the previous step satisfies these conditions, the following Fig. 3 is presented.

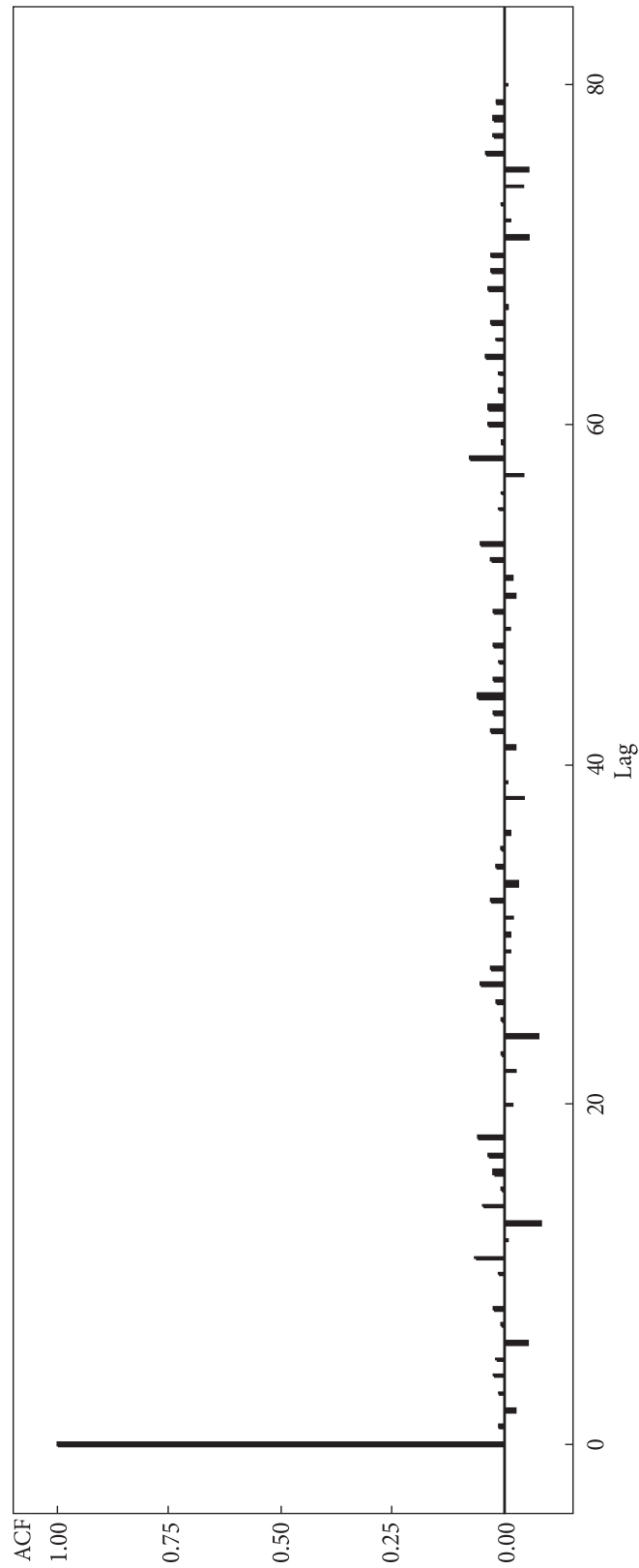
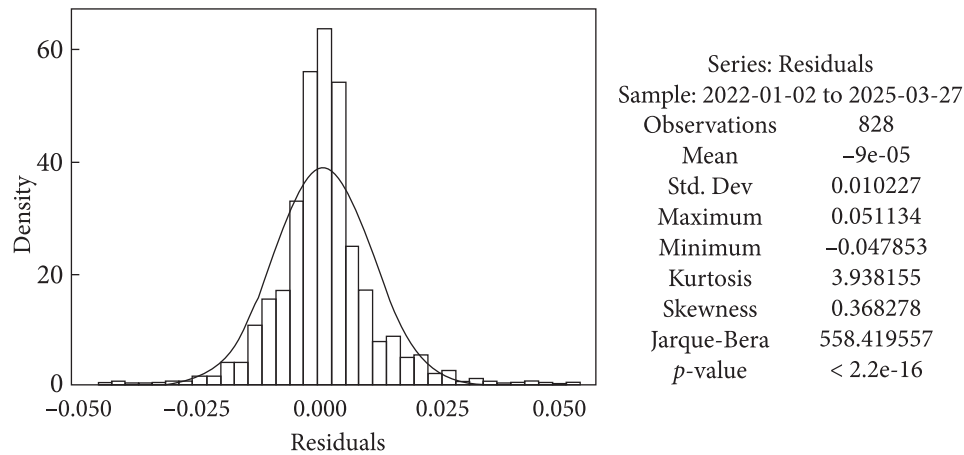


Fig. 4. ACF of Model Residuals
Source: prepared by the authors based on outputs from the *R*.

**Fig. 5.** Residual Series Normality Test Results

Source: prepared by the authors based on outputs from the R.

Table 4. Ljung-Box Test Results

Test	Q-Statistic	Df	p-value
Ljung-Box	14.1846	13	0.3610
	25.8901	24	0.3588
	50.5372	58	0.7462
	82.1410	88	0.6559

Source: prepared by the authors based on outputs from the R.

Table 5. Results of the ARCH Test for Model Residuals

Test	LM	Df	p-value
ARCH Test	70.2654	13	7.172603e-10
	75.0914	24	3.610408e-07
	125.4246	58	7.143073e-07
	163.6893	88	1.759145e-06

Source: prepared by the authors based on outputs from the R.

Fig. 3 illustrates that all the inverse roots of the $AR(p)$ and $MA(q)$ polynomials lie within the unit circle. Consequently, the proposed $ARIMA(2,1,2)$ model is both stable and invertible.

Residual Analysis. Model adequacy requires that residuals satisfy the standard assumptions of the stochastic error term (ξ_t) . The following diagnostic tests are therefore applied.

Test for Autocorrelation in Model Residuals. To verify absence of autocorrelation among the residuals, Fig. 4 presents autocorrelation function (ACF) plot of the residual series.

Fig. 4 shows that the autocorrelation coefficients of the residuals lie within the confidence bounds, indicating **no significant autocorrelation**. This result is confirmed by the Ljung-Box test, where all p -values exceed 0.05 (Table 4).

Normality Test of Model Residuals. To assess the normality of the ARIMA(2,1,2) residuals, Fig. 5 shows a distribution with a sharper peak and heavier tails than a normal distribution. The residuals exhibit leptokurtosis (kurtosis = 3.9381) and slight right skewness (skewness = 0.3682). The Jarque-Bera Test ($p = 2.2 \times 10^{-16}$) rejects the null hypothesis of normality. These results indicate that the residuals are asymmetric, heavy-tailed, and display heteroscedasticity.

ARCH Effect Test. Table 5 shows that all ARCH test p -values are below 0.05, indicating the presence of heteroskedasticity. Therefore, ARCH-type models are applied to model the conditional variance.

SYMMETRIC AND ASYMMETRIC VOLATILITY MODELING

To address the issue of heteroscedasticity in the residuals, volatility was modeled using both symmetric and asymmetric GARCH-type models. The main results obtained from the estimation process are presented in Table 6.

Table 6. Comparison of GARCH-Type Models for Selecting the Best

Models	Model Selection Criteria		
	BIC	Akaike	Hannan-Quinn
apARCH(1,0) — norm	−6.2970	−6.3426	−6.3251
NGARCH(2,0) — snorm	−6.3717	−6.4287	−6.4068
TGARCH(2,0) — M_{powr2} — norm	−6.3722	−6.4292	−6.4073
csGARCH(0,1) — snorm	−6.4808	−6.5321	−6.5125
FIGARCH(3,3) — snorm	−5.5414	−5.6155	−5.5871
eGARCH(0,3) — std	−6.4778	−6.5291	−6.5094
eGARCH(0,3) — snorm	−6.3153	−6.3666	−6.3470
eGARCH(0,2) — snorm	−6.2695	−6.3151	−6.2976
eGARCH(0,1) — snorm	−6.2776	−6.3175	−6.3022
sGARCH(1,0) — snorm	−6.3318	−6.3717	−6.3564
sGARCH(2,0) — M_{powr1} — norm	−6.3513	−6.3968	−6.3794
sGARCH(2,0) — M_{powr1} — snorm	−6.3409	−6.3922	−6.3725
sGARCH(1,0) — M_{powr1} — sged	−2.5213	−2.5726	−2.5529
sGARCH(1,0) — M_{powr2} — sged	−2.5175	−2.5688	−2.5491
sGARCH(1,0) — M_{powr2} — norm	2.6973	2.6574	2.6727

Note: The row in bold indicates the selected optimal model, ARIMA(2,1,2)-CS-GARCH(0,1), which achieved the lowest values across the AIC, BIC, and HQ information criteria among all estimated specifications.

Source: prepared by the authors based on outputs from the *R*.

VOLATILITY MODEL BASED ON INFORMATION CRITERIA

To identify the volatility model, a range of symmetric and asymmetric GARCH specifications with different distributions were estimated. Among more than 750 models, the ARIMA(2,1,2)-CS-GARCH(0,1) with a skewed normal distribution (snorm) provided the best fit, minimizing selection criteria and yielding significant parameters. The model also eliminates residual autocorrelation and heteroskedasticity (Table 7).

Based on the empirical estimation results presented in Table 7, the mathematical formulation of the corresponding optimal statistical model, ARIMA(2,1,2)-CS-GARCH(0,1), can be expressed as follows¹.

Mean Equation: ARIMA (2,1,2)

$$\begin{aligned} \Delta(LSAOCT)_t = & 1.3064 \cdot \Delta(LSAOCT)_{t-1} - \\ & - 0.5801 \cdot \Delta(LSAOCT)_{t-2} - 1.1356 \cdot \varepsilon_{t-1} + 0.3847 \cdot \varepsilon_{t-2} + \zeta_t. \end{aligned} \quad (18)$$

Conditional Variance Equation: CS-GARCH(0,1)

This equation represents the variance dynamics of the ARIMA(2,1,2)-CS-GARCH(0,1) model, capturing both long-term persistence and short-term fluctuations in the volatility of the time series.

$$\begin{cases} h_t = q_t + 0.1573 \cdot (\zeta_{t-1}^2 - q_{t-1}) + 0.1573 \cdot (h_{t-1} - q_{t-1}), \\ q_t = 0.000003 + 0.9797 \cdot q_{t-1}, \\ \zeta_t = \text{skew} - \text{Normal}(0, h_t, \text{Skewness} = 1.0535). \end{cases} \quad (19)$$

The ARIMA(2,1,2)-CS-GARCH(0,1) model captures both mean and variance dynamics. The long-term persistence parameter (0.9797) indicates highly persistent shocks, while short-term shocks are transitory and consistent with mean reversion. Diagnostic tests on standardized residuals (z_t) confirm independence and homoscedasticity (Table 8).

Table 8 shows that the p -values of both ARCH and WLB tests exceed 0.05, indicating no autocorrelation or heteroskedasticity in the residuals. This confirms the adequacy of the ARIMA (2,1,2)-CS-GARCH(0,1) model.

In addition, Fig. 6 shows a close alignment between squared residuals and the estimated conditional variance, indicating that the model accurately captures volatility dynamics across both calm and turbulent periods.

Predictive Performance Evaluation of the ARIMA-CS-GARCH Model. The ultimate goal of time series analysis is to develop models that provide accurate forecasts for informed decision-making. Since the ARIMA(2,1,2)-CS-GARCH(0,1) model demonstrated the best fit-outperforming other models with significant pa-

¹ It is worth noting that the long-term component in a CS-GARCH model is usually presented in R as an AR(1)-type process. $\varphi(\zeta_{t-1}^2 - h_{t-1})$. This is because the software incorporates the innovation term within the parameter estimation procedure, without explicitly displaying it in the output. Therefore, the difference between the theoretical formulation as proposed by R. Engle & G. Lee (1999) and the practical R output does not imply any inconsistency in the model; rather, it reflects a difference in presentation between the theoretical structure and the programming implementation.

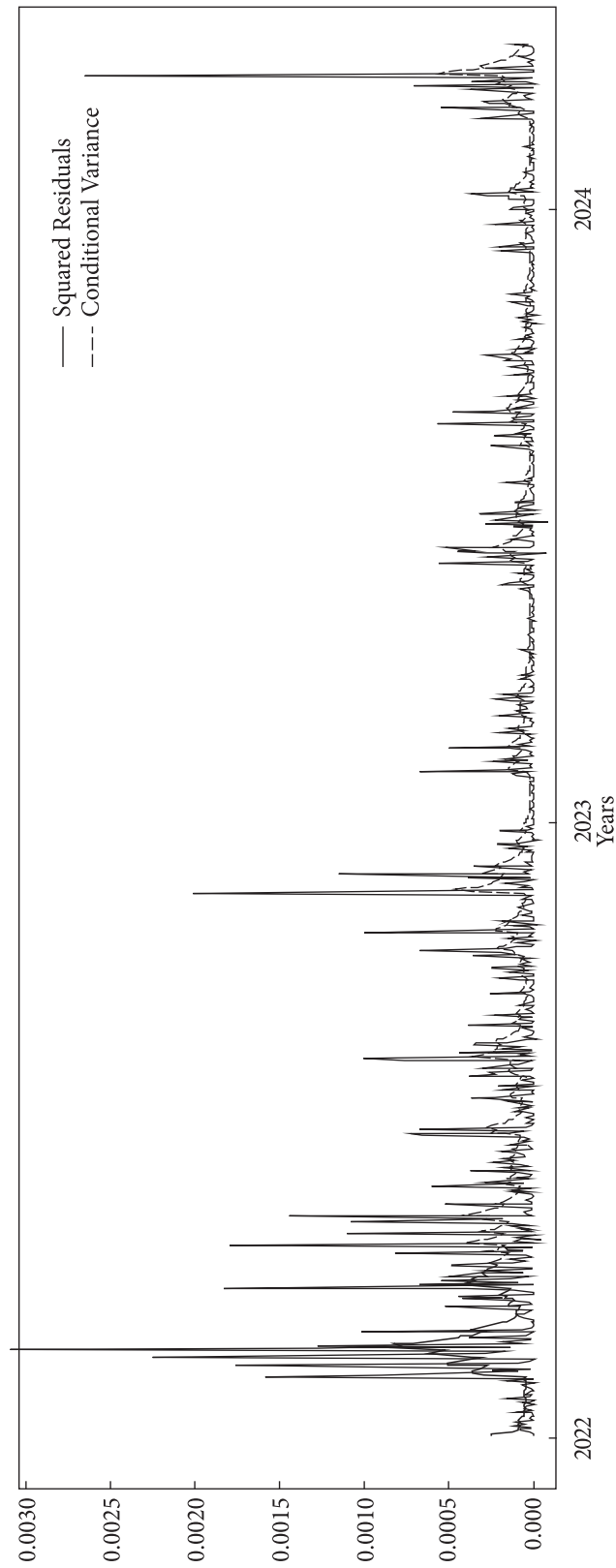


Fig. 6. Comparison of Squared Residuals and Conditional Variance
Source: prepared by the authors based on outputs from the *R*.

Table 7. Estimation Results of the ARIMA (2,1,2)-CS-GARCH(0,1)

Variable	Coefficient	Std.error	T-statistic	p-value
AR(1)	0.358036	0.096170	3.7230	0.00019
AR(2)	0.381348	0.180469	2.1131	0.03459
MA(1)	-0.237436	0.078672	-3.0180	0.00254
MA(2)	-0.523388	0.168320	-3.1095	0.00187
ω	0.000003	0.000000	9961.1048	0.00000
β_1	0.157296	0.003333	47.1875	0.00000
η_{11}	0.979723	0.001404	697.8064	0.00000
η_{21}	0.157296	0.018043	8.7177	0.00000
Skew	1.053543	0.036528	28.8420	0.00000

Source: prepared by the authors based on outputs from the R.

Table 8. Diagnostic Tests for Standardized Residuals

Test	Stat. Val.	Df	p-value
ARCH-Test [Lag 4]	1.5340	4	0.5529
Weighted Ljung-Box-Test[Lag19]	10.479	4	0.3909

Source: prepared by the authors based on outputs from the R.

Table 9. Forecasted Values of the Test Set and Relative Forecast Errors

Date	Actual	ARIMA(2,1,2)		ARIMA(2,1,2)-CS-GARCH(0,1)	
		Predicted	Relative error, %	Predicted	Relative error, %
2025-05-04	24.940	25.1792	0.9590	25.2122	1.0914
2025-05-05	24.900	25.2031	1.2173	25.2239	1.3015
2025-05-06	24.980	25.2465	1.0668	25.2328	1.0126
2025-05-07	24.840	25.2893	1.8090	25.2404	1.6119
2025-05-08	24.840	25.3201	1.9330	25.2466	1.6372

Source: prepared by the authors based on outputs from the R.

Table 10. Predictive Accuracy: ARIMA-CS-GARCH vs ARIMA

Models	Out-of-Sample Data			
	MSE	RMSE	MAE	MAPE
ARIMA(2,1,2)	0.1305	0.3612	0.3477	1.4 %
ARIMA(2,1,2)-CS-GARCH(0,1)	0.1137	0.3373	0.3312	1.33 %

Source: prepared by the authors based on outputs from the R.

rameters and strong diagnostic results — it is used to forecast the next five observations, as summarized in Table 9.

To assess the model's predictive accuracy, the forecasted values from the ARIMA-CS-GARCH model were compared with the actual values in the test set (SAOCTS). The results are summarized in the following Table 10.

Table 10 shows that the ARIMA(2,1,2)-CS-GARCH(0,1) model outperforms the conventional ARIMA(2,1,2) model on the test set (SAOCTS), achieving lower MAE, RMSE, MSE, and MAPE values. This improvement reflects the hybrid model's ability to capture volatility and conditional heteroscedasticity, making its forecasts more accurate and reliable than the purely linear ARIMA model.

RESULTS

This study analyzes the daily adjusted closing prices of Saudi Aramco (SAOC) over the period from January 2, 2022 to May 8, 2025, comprising 834 observations obtained from Yahoo Finance. The dataset was split into a training sample (SAOCT, 829 observations, approximately 99% of the data) and a testing sample (SAOCTS, 5 observations). The training set was used for model estimation and parameter optimization, while the testing set was reserved for out-of-sample forecast evaluation.

Initial graphical analysis indicates that the SAOC price series exhibits pronounced volatility clustering and high variability over time. The series also displays characteristics consistent with a random walk process, where both the mean and variance are time-dependent, suggesting potential non-stationarity. To formally assess stationarity, the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were applied to the training sample. The results confirm the presence of a unit root at the level form. However, after applying a logarithmic transformation followed by first differencing, the series becomes stationary and suitable for econometric modeling.

Based on model estimation results, the ARIMA(2,1,2) specification is identified as the best-fitting model for the conditional mean equation. Nevertheless, diagnostic tests on the residuals reveal significant heteroskedasticity, indicating the presence of ARCH effects and justifying the use of GARCH-type models for volatility modeling. A comprehensive evaluation of more than 750 symmetric and asymmetric GARCH-type models across multiple probability distributions reveals that the CS-GARCH(0,1) model with a skewed normal distribution (snorm) provides the best overall performance in capturing conditional variance dynamics.

The estimated CS-GARCH model decomposes volatility into two distinct components. The first is a long-term component characterized by high persistence, reflecting structural and macroeconomic influences on volatility. The second is a short-term component that captures temporary fluctuations resulting from immediate market shocks, which tend to dissipate relatively quickly. This decomposition provides a more detailed and realistic representation of volatility dynamics compared to standard GARCH models. Finally, the forecasting evaluation demonstrates that the hybrid ARIMA(2,1,2)-CS-GARCH(0,1) model significantly outperforms the standalone ARIMA model in terms of predictive accuracy. This is evidenced by

lower values of key error metrics, including Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Mean Squared Error (MSE).

Overall, the results confirm that integrating ARIMA with CS-GARCH improves both models fit and forecasting performance, making the hybrid framework more suitable for capturing the complex dynamics of Saudi Aramco stock volatility.

DISCUSSION

The empirical findings of this study confirm and extend the existing literature on hybrid ARIMA-GARCH modeling in financial markets. The superior performance of the CS-GARCH(0,1) specification is consistent with B. Abu Zarour & C. Siriopoulos (2008), who highlight its suitability for the Saudi market due to persistent long-term structural shocks. Using a more recent dataset and an expanded distributional framework, this study further validates the robustness of the CS-GARCH approach in capturing volatility dynamics over time. In line with C. Driksaki (2018), K. Sun (2017), and K. Macharia et al. (2025), the results also confirm that incorporating conditional heteroskedasticity significantly improves forecasting performance relative to standalone ARIMA models. However, this study extends prior research by explicitly decomposing volatility into long-term and short-term components, offering a more detailed representation of market risk structure. Methodologically, the study contributes by providing the first empirical application of the ARIMA-CS-GARCH framework to Saudi Aramco using recent high-frequency data up to May 2025. It also conducts a comprehensive model comparison across more than 750 specifications and multiple probability distributions, ensuring robust model selection. In addition, it evaluates out-of-sample forecasting performance, enhancing the practical relevance of the results for investors and risk managers. Overall, these results distinguish the present study from previous research that has primarily focused on standard GARCH specifications, limited datasets, or in-sample analysis. The proposed framework integrates volatility decomposition, extensive model evaluation, and predictive assessment within a unified empirical structure.

Nevertheless, the study has some limitations. It does not incorporate macroeconomic variables such as oil price fluctuations, OPEC+ decisions, or global financial uncertainty, which may influence Saudi Aramco's stock dynamics. Moreover, the relatively small size of the out-of-sample sample limits broader generalization. Future research may extend this framework using multivariate GARCH models, regime-switching approaches, or applications to other Saudi equities.

CONCLUSION AND RECOMMENDATIONS

This study examines the daily adjusted closing prices of Saudi Aramco (SAOC) over the period from January 2, 2022 to May 8, 2025 using a hybrid ARIMA(2,1,2)-CS-GARCH(0,1) framework. The model is estimated on 829 observations and evaluated using a hold-out test sample.

The results indicate that the proposed hybrid specification consistently outperforms the conventional ARIMA model across standard forecasting accuracy measures. In addition, the CS-GARCH structure provides a richer representation of volatility dynamics by decomposing it into long-term and short-term components, thereby offering deeper insight into the behavior of the series. These findings have important practical implications. For investors and portfolio managers, distinguishing between persistent structural volatility and short-term fluctuations improves risk assessment and supports more informed investment timing decisions. For policymakers and regulators, the evidence of strong persistence in the long-term volatility component suggests that effective market stabilization policies should primarily address structural rather than transitory factors.

Overall, the proposed ARIMA-CS-GARCH framework represents a robust and effective tool for modeling and forecasting volatility in the Saudi equity market. Future research may extend this framework by incorporating macroeconomic variables, adopting multivariate specifications such as DCC-GARCH models, and applying the approach to other major stocks or sectors to assess its generalizability under different market conditions.

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REFERENCES

- Dritsaki, C. (2018). The Performance of Hybrid ARIMA-GARCH Modeling and Forecasting Oil Price. *International Journal of Energy Economics and Policy*, 8(3), 14—21. URL: <https://www.econjournals.com/index.php/ijee/article/view/6437>
- Sun, K. (2017). Equity Return Modeling and Prediction Using Hybrid ARIMA-GARCH Model. *International Journal of Financial Research*, 8(3), 154—161. <https://doi.org/10.5430/ijfr.v8n3p154>
- Vo, N., Slepaczuk, R. (2022). Applying Hybrid ARIMA-SGARCH in Algorithmic Investment Strategies on S&P500 Index. *Entropy*, 24(2), 158. <https://doi.org/10.3390/e24020158>
- Hossain, A., Kamruzzaman, Md., Ayub, A. (2015). ARIMA With GARCH Family Modeling and Projection on Share Volume of DSE. *Economics World*, 3(7-8), 171—184. URL: <https://www.davidpublisher.com/Public/uploads/Contribute/5600cb984669e.pdf>
- Adegboyo, O., Sarwar, K. (2025). Modelling and forecasting of Nigeria stock market volatility. *Future Business Journal*, 11(124), 1—13. URL: <https://link.springer.com/article/10.1186/s43093-025-00536-4>
- Abu Zarour, B., Siriopoulos, C. (2008). Transitory and Permanent Volatility Components: The Case of the Middle East Stock Markets. *Review of Middle East Economics and Finance*, 4(2), 1—13. <https://doi.org/10.2202/1475-3693.1060>
- Liu, T., Shi, Y. (2022). Innovation of the Component GARCH Model: Simulation Evidence and Application on the Chinese Stock Market. *Mathematics*, 10(11), 1903. <https://doi.org/10.3390/math10111903>

- Contreras-Reyes, J., Zavala, J., Idrovo-Aguirre, B. (2024). Analyzing the Selective Stock Price Index Using Fractionally Integrated and Heteroskedastic Models. *Journal of Risk and Financial Management*, 17(9), 401. <https://doi.org/10.3390/jrfm17090401>
- Benbachir, S. (2025). Assessing informational efficiency in largest African stock markets by modeling dual long memory: An ARFIMA-FIGARCH approach. *Investment Management and Financial Innovations*, 22(2), 238—253. [http://dx.doi.org/10.21511/imfi.22\(2\).2025.19](http://dx.doi.org/10.21511/imfi.22(2).2025.19)
- Wanjuki, T., Lumumba, V., Kimtai, E., Mbaluka, M., Njoroge, E. (2024). Comparative Analysis of GARCH-Based Volatility Models of Financial Market Volatility: A Case of Nairobi Security Market PLC, Kenya. *European Journal of Mathematics and Statistics*, 5(4), 1—18. <https://doi.org/10.24018/ejmath.2024.5.4.310>
- Karugano, R., Kariuki, S., Kariuki, P. (2023). Predictability of GARCH-Type Models in Estimating Stock Returns Volatility. Evidence from Kenya. *International Journal of Professional Business Review*, 8(11), e02813. <https://doi.org/10.26668/businessreview/2023.v8i11.2813>
- Wang, Y., Xiang, Y., Lei, X., Zhou, Y. (2022). Volatility Analysis Based on GARCH-Type Models: Evidence from the Chinese Stock Market. *Economic Research — Ekonomska Istraživanja*, 35(1), 2530—2554. <https://doi.org/10.1080/1331677X.2021.1967771>
- Xiang, Y. (2022). Using ARIMA-GARCH Model to Analyze Fluctuation Law of International Oil Price. *Mathematical Problems in Engineering*, 1—7. <https://doi.org/10.1155/2022/3936414>
- Singh, R., Singh, Y., Kumar, S., Kumar, A., Alruwaili, W. (2024). Mapping Risk—Return Linkages and Volatility Spillover in BRICS Stock Markets through the Lens of Linear and Non-Linear GARCH Models. *Journal of Risk and Financial Management*, 17(10), 437. <https://doi.org/10.3390/jrfm17100437>
- Sözen, Ç. (2025). Volatility dynamics of cryptocurrencies: a comparative analysis using GARCH-family models. *Future Business Journal*, 11, 166. URL: <https://link.springer.com/article/10.1186/s43093-025-00568-w>
- Queiroz, R., David, S. (2023). Performance of the Realized-GARCH Model against Other GARCH Types in Predicting Cryptocurrency Volatility. *Risks*, 11(12), 211. <https://doi.org/10.3390/risks11120211>
- Silva, L., Maciel, L. (2025). Cryptocurrency price returns volatility modeling and forecasting with GARCH models. *RAUSP Management Journal*, 60(01). <https://doi.org/10.1108/RAUSP-04-2023-0056>
- Box, G., Jenkins, G., Reinsel, G., Ljung, G. (2016). Time Series Analysis: Forecasting and Control. 5th ed. Hoboken, John Wiley & Sons. 720 p. URL: <https://bit.ly/4b5bDzf>
- Wang, Y. (2024). Application of ARIMA-GARCH-S model to short-term stock forecasting. *Finance & Economics — Dean & Francis Press.*, 1(4), 1-10. <https://doi.org/10.61173/y2k63978>
- Engle, R., Lee, G. (1999). A long-run and short-run component model of stock return volatility. In: R. Engle, H. White, O. Press (Ed.). Cointegration, causality, and forecasting: A festschrift in honour of Clive W.J. Granger. P. 475—497. Oxford. <https://doi.org/10.1093/oso/9780198296836.003.0020>
- Tsay, R. (2010). Analysis of Financial Time Series. 3rd ed. Hoboken, John Wiley & Sons. <https://doi.org/10.1002/9780470644560>
- Nelson, D., Cao, C. (1992). Inequality constraints in the univariate GARCH model. *Journal of Business & Economic Statistics*, 10(2), 229—235. <https://doi.org/10.1080/07350015.1992.10509902>
- Nelson, D. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, 59(2), 347—370. <https://doi.org/10.2307/2938260>

- Glosten, L., Jagannathan, R., Runkle, D. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, 48(5), 1779—1801. <https://doi.org/10.1111/j.1540-6261.1993.tb05128.x>
- Engle, R., Ng, V. (1993). Measuring and Testing the Impact of News on Volatility. *The Journal of Finance*, 48(5), 1749—1778. <https://doi.org/10.1111/j.1540-6261.1993.tb05127.x>
- Ding, Z., Granger, C., Engle, R. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, 1(1), 83—106. [https://doi.org/10.1016/0927-5398\(93\)90006-D](https://doi.org/10.1016/0927-5398(93)90006-D)
- Baillie, R., Bollerslev, T., Mikkelsen, H. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 74(1), 3—30. [https://doi.org/10.1016/S0304-4076\(95\)01749-6](https://doi.org/10.1016/S0304-4076(95)01749-6)
- Macharia, K., Atitwa, E., Mugo, D., Kawira, M. (2025). Modeling stock price trends and volatility in emerging markets using ARIMA and GARCH approaches. *International Journal of Advanced and Applied Sciences*, 12(7), 134—143. <https://doi.org/10.21833/ijaas.2025.07.013>

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ПОРІВНЯЛЬНЕ ЕКОНОМЕТРИЧНЕ ДОСЛІДЖЕННЯ МОДЕЛЕЙ ARIMA-GARCH ДЛЯ МОДЕЛЮВАННЯ ВОЛАТИЛЬНОСТІ АКЦІЙ SAUDI ARAMCO

Досліджено динаміку волатильності щоденних скоригованих цін закриття акцій Saudi Aramco (SAOC) за допомогою гібридної економетричної моделі, що поєднує моделі ARIMA і CS-GARCH. Фінансові часові ряди зазвичай характеризуються кластеризацією волатильності, умовною гетероскедастичністю і асиметричними реакціями на шоки, що обмежує ефективність суто лінійних моде-

лей. Для вирішення цих проблем застосовано модель ARIMA(2,1,2) для фіксації лінійних залежностей умовного середнього і специфікацію CS-GARCH (0,1) для моделювання змінної у часі умовної дисперсії та розкладання волатильності на коротко- й довгострокові компоненти.

З використанням даних з 2 січня 2022 р. до 8 травня 2025 р. (834 спостереження) показано, що ряд є інтегрованим першого порядку і демонструє значну гетероскедастичність. Після порівняння моделей, яке охоплювало понад 750 специфікацій типу GARCH, модель ARIMA(2,1,2)-CS-GARCH(0,1) з асиметричним нормальним розподілом було визначено як оптимальну специфікацію на основі інформаційних критеріїв і діагностичних тестів. Результати засвідчують високу стійкість компонента довгострокової волатильності, тоді як короткострокові шоки демонструють тимчасові ефекти, що поступово зникають.

Результати прогнозування поза вибіркою підтверджують, що гібридна модель перевершує традиційну модель ARIMA за всіма показниками точності (MAPE, RMSE, MAE й MSE). Зроблено висновок, що гібридні моделі ARIMA-CS-GARCH пропонують робастний і надійний інструментарій для моделювання і прогнозування волатильності акцій на високодинамічних ринках, надаючи цінну інформацію для інвесторів, портфельних менеджерів і аналітиків ризиків на фінансовому ринку Саудівської Аравії.

Ключові слова: волатильність акцій; Saudi Aramco; моделі ARIMA-GARCH; фінансові ринки.

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