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Modeling subsurface and surface runoff during heavy rainfall on sloping land

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The fundamental principles of storm water runoff theory and the mathematical problem of combined surface runoff and rainwater filtration on sloping agricultural land in a hydraulic statement are formulated. An approximate solution to the problem has been obtained and justified. Based on this, a method has been developed and illustrated for the engineering calculation of local and integral hydrological and filtration characteristics for the first stage of surface and saturated flow formation (surface flooding). Recommendations are given for estimating the volumes of productive (soil moisture) and unproductive (surface runoff) water for the first and, in part, the second (drainage) stages. With regard to the climatic and soil-hydrophysical conditions of the western region of Ukraine, the reality of the significant influence of capillary forces on infiltration and, in general, on the water-physical situation on a sloping site is shown.

Keywords: *modeling, infiltration, rainfall, slope, head losses, surface runoff, engineering method.*

Introduction. Intense rainfall events on sloping agricultural land often lead to a complex redistribution of precipitation between surface runoff and subsurface infiltration. Unlike flat terrains, even relatively short periods of heavy rain on slopes may result in substantial surface flow, limiting the effective use of atmospheric water by crops and altering local soil moisture regimes.

In today's conditions of increasing moisture deficiency in soil ecosystems in areas of intensive agricultural production, the rational use of water resources, which are generally limited, is becoming extremely important. In order to make the most of the increasingly frequent intense prolonged rainfall for the benefit of cultivated crops, surface water should be retained and subsequently used for irrigation purposes.

When designing appropriate storage facilities, it is necessary to proceed from reliable data on atmospheric water resources and the patterns in the precipitation distribution between surface and subsurface runoff. Such information can only be obtained systematically and in advance through the widespread use of mathematical modeling methods. The models are particularly important for engineering applications related to water retention, erosion control, and estimation of productive

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and unproductive components of rainfall that describe the coupled dynamics of surface flow and infiltration into initially unsaturated soils. Despite the large number of existing studies devoted to rainfall-runoff processes on hillslopes, many practical calculations are still based on either purely surface or purely subsurface approaches, which restricts their applicability under flooding conditions. Given the exceptional significance of the aforementioned models and the corresponding theoretical developments based on them for a variety of engineering applications, a huge number of publications have been devoted to them and their information support. We will limit ourselves to mentioning only a few characteristic diverse works [1—5]. At the same time, it is necessary to highlight the long-standing fundamental work of Professor A.N. Befani, which systematically presents the results of research on this problem using rigorous analytical methods, as well as experimental methods at numerous experimental sites [6].

The **main objective** of this article is to provide a generalized assessment of the consequences of heavy precipitation for the regulated water regime on sloping land areas. A coupled hydraulic—filtration framework is adopted to analyze the first stage of heavy rainfall on sloping land, when surface flooding and intensive soil wetting occur simultaneously. Based on a simplified but physically justified formulation of the governing equations, an approximate analytical solution is derived and used to develop an engineering method for estimating local and integral hydrological characteristics. Particluar attention is paid to the role of soil moisture conditions and capillary forces, which may significantly affect infiltration rates and, consequently, the balance between productive soil moisture and surface runoff.

Results and discussion. In accordance with modern concepts of combined surface and infiltration water flow, it is appropriate to represent the fundamental mathematical model in the form of two interconnected compartments: hydrodynamic and filtration. The first compartment describes, in a hydraulic approximation, the formation of a surface layer of water, which actually moves under the action of two main forces and has a significant impact on the wetting of the porous medium. Previous detailed analysis of the general equation of motion has allowed us to identify gravity and resistance as the dominant forces [7]. If the surface is flat, then the specified equation and the continuity equation applicable to the surface flooding phase (I) take the following form [6, 8, 9]

$$\sin\psi - \frac{dh_f}{dl} = 0, \tag{1}$$

$$\frac{\partial Q}{\partial l} + \frac{\partial h_w}{\partial t} = \varepsilon - V_i. \tag{2}$$

Here ψ is the surface slope, h_f are the head losses due to friction on the slope section under consideration, Q is the specific discharge (per unit width of the surface flow), h_w is the water level at the ground surface, V_s is the average velocity of the surface water, l is the coordinate along slope, ε is the constant (average) rainfall intensity for the calculation period, V_i is the infiltration rate, g is the free fall acceleration,

Since the porous medium is initially unsaturated, i.e., its moisture content is less than its total moisture capacity, and the water levels on the soil surface (h_w) and inside it (Z_w) are measured from it, a homogeneous initial condition is added to system (1), (2) in the case of stage I.

$$t = 0,\ h_w = 0. \tag{3}$$

Mechanical energy losses due to viscous forces on different types of natural and cultivated surfaces have been thoroughly studied using experimental methods. An overview of the relevant results is presented, for example, in [6, 10]. It has been established that head losses along the length in such cases are determined by the average velocity and the height of the water layer on the surface. The corresponding formula for specific losses applicable to flat, bumpy bare and grass-covered surfaces, is usually taken into account in (3) in such generalized forms

$$\frac{dh_f}{dl} = \lambda V^{\alpha} h_w^{\beta} = \lambda Q^{\alpha} h_w^{\beta-\alpha}, \tag{4}$$

where λ, α, β are empirical constants. The drag coefficient λ can vary significantly depending on surface roughness, the presence of vegetation, and the phase of vegetation development. Unlike λ, exponents α, β are conservative, and it is often justified to assume $\alpha = 2$, $\beta = -2$ due to the turbulent nature of surface flow.

The formation of a layer of water on the soil surface causes a corresponding increase in infiltration rates and the movement of the saturation (wetting) front. It is reasonable to assess this effect based on the filtration compartment. When formulating it, the constancy of moisture in the aeration zone and the uniformity of the soil are assumed. Then the specified compartment includes the filtration equation in the form [11, 12]

$$n_e \frac{dZ_w}{dt} = -k \frac{Z_w + \psi l - h_w(t) + z_w}{Z_w + \psi l}, \tag{5}$$

which is valid for small ψ and is supplemented for the first stage by the initial condition

$$t = 0, \quad Z_w = -\psi l. \tag{6}$$

Here n_s is the effective porosity (constant difference between the total and actual moisture capacity, taking into account trapped air), Z_w is the saturation front coordinate (coordinate axis is directed vertically upward with its origin at the ground surface, so that $Z_w \le 0$), k is the hydraulic conductivity; z_w is the increase in pressure at the saturation (wetting) front due to capillary forces [13—15].

An approximate solution to the problem of joint rainwater runoff down a slope and its intensive absorption by the soil was found in stages. In the first stage, the water exchange function between the surface flow and adjacent environments (atmosphere, soil) $\varepsilon - V_i(t, l)$ was assumed to be constant and equal to $\tilde{\varepsilon} = \varepsilon - k$. It was this value $\tilde{\varepsilon}$ that appeared in equation (2) instead of the variable quantity $\varepsilon - V_i$. Then the system (1), (2) was reduced to an equation, for example, with respect to h_w

$$\frac{\alpha - \beta}{\alpha} \left(\frac{\psi}{\lambda} \right)^{\frac{1}{\alpha}} h_w^{-\frac{\beta}{\alpha}} \frac{\partial h_w}{\partial l} + \frac{\partial h_w}{\partial t} = \tilde{\varepsilon}. \tag{7}$$

The exact solution of equation (7) satisfying the homogeneous boundary conditions, namely (3) and

$$l = 0, \quad Q = 0, \tag{8}$$

represents the following continuous function [6]

$$h_w(l,t)=\begin{cases} h_{w\infty}(l), & \text{at } l\le l_0(t),\\ \tilde{\varepsilon}t, & \text{at } l>l_0(t),\end{cases} \tag{9}$$

where

$$h_{w\infty}(l)=\left(\frac{\lambda}{\psi}\right)^{\frac{1}{\alpha-\beta}}(\tilde{\varepsilon}l)^{\frac{\alpha}{\alpha-\beta}}. \tag{10}$$

Its derivative diverges at $l=l_0(t)$, where

$$l_0(t)=\left(\frac{\psi}{\lambda}\right)^{\frac{1}{2}}\tilde{\varepsilon}t^2. \tag{11}$$

In general, this solution is obtained for the more general case $\tilde{\varepsilon}=\tilde{\varepsilon}(t)$. However, neither it nor the particular representation (9) essentially allow us to estimate the total volumes of productive and unproductive moisture to be made.

It was relatively easy to develop an engineering method for separately calculating the volumes of rainwater infiltrating and flowing down the slope by using average characteristics. First of all, the quantities h_w and V_i were averaged within the considered section on the slope $L\ge l\ge 0$, so that

$$h_{av}(t)=\frac{1}{L}\int_0^L h_w(t,l)\,dl,\ V_{ia}(t)=\frac{1}{L}\int_0^L V_i(l,t)\,dl. \tag{12}$$

Then equation (2) at average rate V_{ia} was simplified as follows

$$\frac{\partial Q}{\partial l}+\frac{dh_{av}}{dt}=\varepsilon-V_{ia}(t). \tag{13}$$

From (13) follows the equation regarding h_{av}

$$\frac{dh_{av}}{dt}=\varepsilon-V_{ia}-\left(\frac{2\alpha-\beta}{\alpha-\beta}\right)^{\frac{\alpha-\beta}{\alpha}}\left(\frac{\psi}{\lambda}\right)^{\frac{1}{\alpha}}\frac{h_{av}^{\frac{\alpha-\beta}{\alpha}}}{L}, \tag{14}$$

which was solved under the initial condition

$$t=0,\ h_{av}=0. \tag{15}$$

For arbitrary values of α, β, and $V_{ia}=V_{ia}(h_{av})$ or $const$, solution (14), (15) is expressed by the inverse integral function

$$t(h_{av})=L\int_0^{h_{av}}\frac{d\zeta}{(\varepsilon-V_{ia})L-\left(\frac{2\alpha-\beta}{\alpha-\beta}\right)^{\frac{\alpha-\beta}{\alpha}}\left(\frac{\psi}{\lambda}\right)^{\frac{1}{\alpha}}\zeta^{\frac{\alpha-\beta}{\alpha}}}. \tag{16}$$

In a common special case $\alpha=-\beta=2$ and $\varepsilon-V_{ia}\approx\hat{\varepsilon}$, the relationship between t and h_{av} can be represented using elementary functions in two equivalent forms, in particular,

$$h_{av}(t)=h_{av\infty}(L)\frac{1-e^{-\gamma t}}{1+e^{-\gamma t}}, \tag{17}$$

where $\gamma=3\left(\frac{\tilde{\varepsilon}}{L}\right)^{\frac{1}{2}}\left(\frac{\psi}{\lambda}\right)^{\frac{1}{4}}$, $h_{av\infty}(L)=\frac{2}{3}\left(\frac{\lambda}{\psi}\right)^{\frac{1}{4}}(\tilde{\varepsilon}L)^{\frac{1}{2}}$. Taking into account the smallness of γ ($\gamma<<1$), expression (20) can be significantly simplified, so that

$$h_{av}(t)=h_{av\infty}(L)\frac{\gamma t}{2}=\tilde{\varepsilon}t\,. \tag{18}$$

Accepting $h_{av}(t)$ according to (18) instead of (17) in the calculations leads to additional errors. The characteristic time t_m, starting from which h_{av} will be calculated with an error ϑ and greater, can be easily determined by fitting from equation

$$(1+\vartheta)h_{av\infty}(1-e^{-\gamma t_m})=\tilde{\varepsilon}t_m(1+e^{-\gamma t_m})\,. \tag{19}$$

The approximate solution presented allows us to easily determine the proportions of rainfall that form subsurface and surface runoff and, ultimately, the volumes of productive and unproductive rainwater. First, the intensity of water infiltration is determined simultaneously across the entire area w_i under consideration as follows

$$w_1(t)=\int_0^L V_i dl=V_{ia}L\,. \tag{20}$$

Here, the average rate V_{ia} is equal to $n_e\frac{d\tilde{Z}_a}{dt}$ and is obtained as a result of a numerical solution to the problem

$$n_e\frac{d\tilde{Z}_a}{dt}=-k\frac{\tilde{Z}_a+z_w-h_{av}(t)}{\tilde{Z}_a};\qquad \text{at}\qquad t=0,\ \tilde{Z}_a=0\,, \tag{21}$$

where $h_{av}(t)$ is taken at $\alpha=-\beta=2$ according to (17) or (18). In a more complicated case of arbitrary values of α,β, it is necessary to use a solution in parametric form, where h_{av} serves as the parameter. The pair of equations linking $\tilde{Z}_a$ and t with h_{av} primarily includes equation (16), to which the transformed equation (21) is added. Here, it is preliminarily assumed that $\varepsilon-V_{ia}=\tilde{\varepsilon}$ and, in fact, a replacement of the independent variable is performed, according to which

$$dt=f(h_{av})dh_{av},$$

where $f(h_{av})=\left[\tilde{\varepsilon}-\left(\frac{2\alpha-\beta}{\alpha-\beta}\right)^{\frac{\alpha-\beta}{\alpha}}\left(\frac{\psi}{\lambda}\right)^{\frac{1}{\alpha}}\frac{h_{av}^{\frac{\alpha-\beta}{\alpha}}}{L}\right]^{-1}$.

Thus, equation (21) is reduced to the form

$$n_e \frac{d\tilde{Z}_a}{dh_{av}} = -kf(h_{av}) \frac{\tilde{Z}_a + z_w - h_{av}}{\tilde{Z}_a} \tag{22}$$

and is solved provided that

$$h_{av} = 0, \quad \tilde{Z}_a = 0 . \tag{23}$$

Thus, the desired solution has a combined analytical-differential form. To establish the quantities $\tilde{Z}_a$, t dependent on the introduced parameter h_{av}, its value is first specified, and then the corresponding time t is found from (16), and the corresponding unique value $\tilde{Z}_a$ is found by numerical solution of (22) and (23). Finally, the value of V_{ia} is calculated as the value of the right-hand side of equation (22) with the opposite sign for the known values of $\tilde{Z}_a$, h_{av}, and corresponds to the previously found t.

If h_{av} depends linearly on t according to (9) or (18), then the relationship between $\tilde{Z}_a$ and t is represented strictly in implicit form. To derive the corresponding analytical expression, first in (5) at h_w according to (18), the dependent and independent variables were replaced

$$Z_w = \tilde{Z}_w - \psi l , \; t = \tilde{t} + \frac{z_w}{\gamma} . \tag{24}$$

Then the new dependent variable was introduced

$$\tilde{Z}_w = \tilde{t}\,\Omega(\tilde{t}) . \tag{25}$$

The problem regarding Ω involved an equation with separable variables and an initial condition in general form

$$\tilde{t} \frac{d\Omega}{d\tilde{t}} = \frac{k\gamma - k\Omega - n_e\Omega^2}{n_e\Omega} , \tag{26}$$

$$\tilde{t} = \tilde{t}_0, \quad \Omega = \Omega_0 . \tag{27}$$

If capillary forces have a significant effect on soil saturation $(z_w < 0)$, then according to (25), the initial condition should be taken as follows $(\Omega_0 = 0)$

$$\tilde{t} = -\frac{z_w}{\gamma}, \quad \Omega = 0 . \tag{28}$$

Then the relationship between Ω and $\tilde{t}$ will be

$$\left(1 - \frac{\Omega}{\omega_1}\right)^{\frac{\omega_1}{\omega_2 - \omega_1}} \left(1 - \frac{\Omega}{\omega_2}\right)^{\frac{\omega_2}{\omega_1 - \omega_2}} = -\frac{z_w}{\gamma \tilde{t}} . \tag{29}$$

Here, the values $\omega_{1,2}=\frac{k}{2n_e}(-1\pm\sqrt{\Delta})$ are the roots of the quadratic equation $n_e\Omega^2+k\Omega-k\gamma=0$, $\Delta=1-\frac{4\gamma n_e}{k}$, $\omega_1-\omega_2=\frac{k}{n_e}\sqrt{\Delta}$. Returning from Ω, $\tilde{t}$ to the previous variables, the following representation was obtained for $Z_w(t)$ in implicit form

$$\left(1-\frac{2n_e\tilde{\varepsilon}(Z_w+\psi t)}{k(\sqrt{\Delta}-1)(\tilde{\varepsilon}t-z_w)}\right)^{\frac{\sqrt{\Delta}-1}{2\sqrt{\Delta}}}\left(1+\frac{2n_e\tilde{\varepsilon}(Z_w+\psi t)}{k(1+\sqrt{\Delta})(\tilde{\varepsilon}t-z_w)}\right)^{\frac{\sqrt{\Delta}+1}{2\sqrt{\Delta}}}=\frac{z_w}{z_w-\tilde{\varepsilon}t}. \tag{30}$$

In a special case of negligible capillary forces $(z_w\approx 0)$ in the initial condition (at t, $\tilde{t}=0$) the value Ω_0 is initially unknown. Then the calculated equation relative to Ω is reduced to the form

$$(\omega_1-\Omega)^{\frac{\omega_1}{\omega_2-\omega_1}}(\omega_2-\Omega)^{\frac{\omega_2}{\omega_1-\omega_2}}=0. \tag{31}$$

Since only the second root has physical meaning, the expression for Z_w should be

$$Z_w(t,l)=-\psi l-\frac{kt}{2n_e}(1+\sqrt{\Delta}). \tag{32}$$

Then the average rate V_{ia} will be constant

$$V_{ia}=k\left(1+\frac{2n_e\gamma}{k(1+\sqrt{\Delta})}\right). \tag{33}$$

The highlighted case appears to be clearly preferable in practice, as there is no need to specify the problematic empirical constant z_w (or possibly variable). However, the saturation front often moves significantly faster than in case $z_w=0$ (33), and therefore it is important to know the actual value of z_w.

By the end of prolonged rainfall $(t=t_*)$, a significant amount of water may accumulate on the surface. If the length of the full-runoff zone covers the entire slope $(l_0(t_*)\geq L)$, then the maximum level $h_{w\infty}$ will be described at $L\geq l\geq 0$ by equation (10). Thus, the maximum volume of surface water W_{wm} will be

$$W_{wm}=\frac{\alpha-\beta}{2\alpha-\beta}\left(\frac{\lambda}{\psi}\right)^{\frac{1}{\alpha-\beta}}\tilde{\varepsilon}^{\frac{\alpha}{\alpha-\beta}}L^{\frac{2\alpha-\beta}{\alpha-\beta}}. \tag{34}$$

The fundamental differences between the second $(t>t_*)$ and first $(t_*\geq t\geq 0)$ stages are, first, the absence of atmospheric feeding of the surface flow, second, the presence of an initial volume of water on the surface, which, despite $\varepsilon=0$ ensures subsequent surface runoff and replenishment of soil moisture reserves, and thirdly, a gradual decrease in both runoff quantities to 0 at the given point in time $(t=t_{**})$, which, unlike the end time of the first stage, must be calculated.

In general, two approaches are justified for establishing the final ratio between the volume of rainwater that has saturated the soil and the volume of the water that has flowed beyond the boundaries of the designated slope area. It is much easier to estimate this ratio based on known total intensities of infiltration (w_i), surface water accumulation (w_w) and its discharge through the lower boundary of the site at the moment of precipitation cessation $(t = t_*)$.

However, the second approach is more reasonable from a physical point of view. Then, when evaporation is neglected, the continuity equation analogous to (13) is of fundamental importance for the second stage

$$\frac{\partial Q_2}{\partial l} + \frac{dh_{av2}}{dt} = -V_{ia2}(t); \quad V_{ia2} > 0 \tag{35}$$

and the initial condition

$$t = t_*, \quad h_{av} = h_{av*}, \tag{36}$$

where h_{av*} is the average water level on the surface at $t = t_*$. Accordingly, the key subject of the calculations is the average surface water level in the area under consideration h_{av2} as a function of t.

In general, the desired h_{av2}, $\tilde{Z}_{a2}$ should be found by solving the system of ordinary equations

$$\frac{dh_{av2}}{dt} = -\left(\frac{2\alpha - \beta}{\alpha - \beta}\right)^{\frac{\alpha-\beta}{\alpha}} \left(\frac{\psi}{\lambda}\right)^{\frac{1}{\alpha}} \frac{h_{av2}^{\frac{\alpha-\beta}{\alpha}}}{L} - k\frac{\tilde{Z}_{a2} + z_w - h_{av2}(t)}{\tilde{Z}_{a2}}, \tag{37}$$

$$\frac{d\tilde{Z}_{a2}}{dt} = -\frac{k}{n_e}\frac{\tilde{Z}_{a2} + z_w - h_{av2}(t)}{\tilde{Z}_{a2}} \tag{38}$$

under conditions (36) and

$$t = t_*, \quad \tilde{Z}_{a2} = \tilde{Z}_a(t_*) = \tilde{Z}_{a*} \tag{39}$$

by numerical methods using modern mathematical analysis software packages. However, another approach can ensure high calculation accuracy with significantly less effort. It involves replacing the variables h_{av2}, V_{ia2} with constant values as follows

$$h_{av2}(t) \approx \frac{h_{av*}}{2}, \; V_{ia2}(t) \approx \frac{V_{ia*}}{2}.$$

Such simplification is justified in view of the almost uniform decrease of h_{av2} from h_{av*} to 0 and V_{ia2} from V_{ia*} to 0. Then the solution of equation (37) under condition (36) will be

$$t(h_{av2}) = t_* + 2L\int_{h_{av2}}^{h_{av*}} \frac{d\zeta}{2\left(\frac{2\alpha-\beta}{\alpha-\beta}\right)^{\frac{\alpha-\beta}{\alpha}}\left(\frac{\psi}{\lambda}\right)^{\frac{1}{\alpha}}\zeta^{\frac{\alpha-\beta}{\alpha}} - LV_{av*}}. \tag{40}$$

In the special case $\alpha = -\beta = 2$, equation (38) reduces to a simpler form

$$\frac{d\tilde{Z}_{a2}}{dt} = -\frac{k}{n_e}\frac{2\tilde{Z}_{a2} + 2z_w - h_{av*}}{\tilde{Z}_{a2}}. \quad (41)$$

Solution (41) under condition (39) is presented in the form of the inverse function

$$t = t_* + \frac{n_e}{k}\left(\tilde{Z}_{a*} - \tilde{Z}_{a2} - \frac{2z_w - h_{av*}}{2}\ln\frac{2\tilde{Z}_{a*} + 2z_w - h_{av*}}{2\tilde{Z}_{a2} + 2z_w - h_{av*}}\right). \quad (42)$$

Thanks to the use of (40) and (42), an important practical value can be easily calculated in three steps, namely, the total specific volume of infiltration water from precipitation with a total volume of $\varepsilon L t_*$. In the first step, according to (40), the time t_{**} is calculated. In the second step, by fitting from (42), the value of $\tilde{Z}_{a2}$ corresponding to t_{**} is found. Finally, the total volume W_i is immediately determined as $L\tilde{Z}_a(t_{**})$.

The purpose of the quantitative analysis, along with illustrating the derived calculation expressions, was also to evaluate possible errors in the calculations associated with the introduction of effective values, the significance of the influence of capillary forces, and, most importantly, to demonstrate the effectiveness of the new method for separately calculating productive (infiltrated into the soil) and unproductive (flowing out) components of intense rainfall falling on a sloping site. The duration of rainfall and the calculation period were taken to be equal to the time of formation of the full-runoff zone on the entire slope under consideration $L \geq l \geq 0$, so that $l_0(t_*) = L$. The following typical values of model coefficients were preliminarily recorded: $\varepsilon = 2 \cdot 10^{-5}$ m/s, $k = 10^{-5}$ m/s (0.865 m/day), $\psi = 0.003$, $n_e = 0.25$. The value $\lambda = 3.46 \cdot 10^{-5}$ s^2 corresponded to a flat, compacted, bare surface [6]. In addition, for a preliminary assessment of the significance of the initial water-physical state of the soil, parameter z_w was continuously varied.

First, the change in water rainfall intensity over time $w_i(t)$ was determined across the entire slope ($L \geq l \geq 0$). Its value, calculated based on (9), was used as a reference for w_i. At the same time, the components of the runoffs within the full-runoff zone and partial runoff zone were determined separately. The monotonic decrease of the calculated curves in Fig. 1 means that the increase in the inlet head due to the accumulation of water on the ground surface does not compensate for the decrease in the head gradient in the filtration field due to its expansion. First of all, it should be noted that the reference (1) and approximate (2) curves are close to each other. Curve 2 is calculated according to (20), (21). It is also important to note that the curves intersect approximately in the middle of the calculation period. The maximum deviation between them occurred at the end of the specified period and was amounted to 5 %. This clearly shows that the integral of $w(t)$, i.e., the total volume of water infiltrated into the soil in 30 minutes, calculated on the basis of exact and approximate solutions, will practically coincide. It is the volume W_i as the most indicative characteristic that became the subject of calculations for the other two series of items.

In the first series, parameter z_w was used as an argument. Fig. 2 shows the plots of the increase in the reduced volume $W_i/(n_e L)$ calculated in accordance with (21) when z_w changes from 0 to -0.01. This interval corresponds to the range of initial moisture content values in the soil aeration zone, which must be determined experimentally. Due to the lack of suitable data on z_w the results presented are hypothetical. However, they convincingly demonstrate the high sensitivity of the hydrological and filtration conditions to the initial water-physical state of the soil. Consequently,

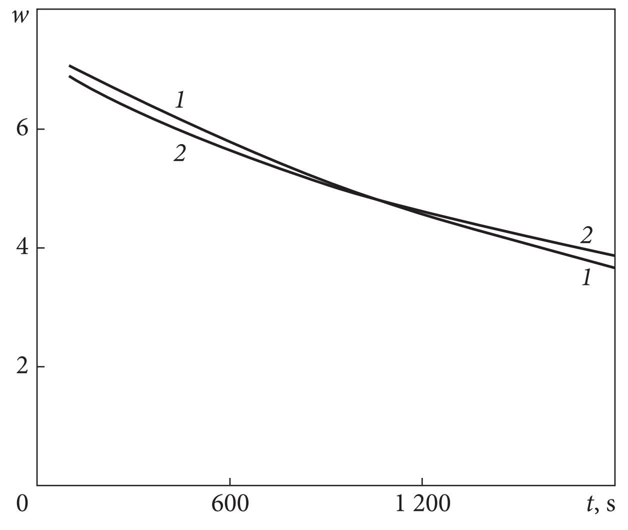

Fig. 1. Change in rainwater infiltration intensity over time under the influence of surface runoff in the calculated site ($L \geq l \geq 0$): *1* — reference; *2* — approximate

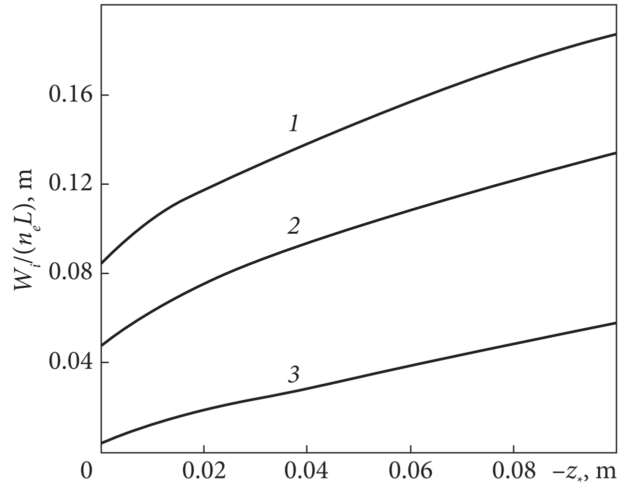

Fig. 2. Dependence of $W_i/(n_e L)$ on $-z_*$: *1* — $t = t_* = 1\,794$ s; *2* — $t = 1\,000$ s; *3* — $t = 100$ s

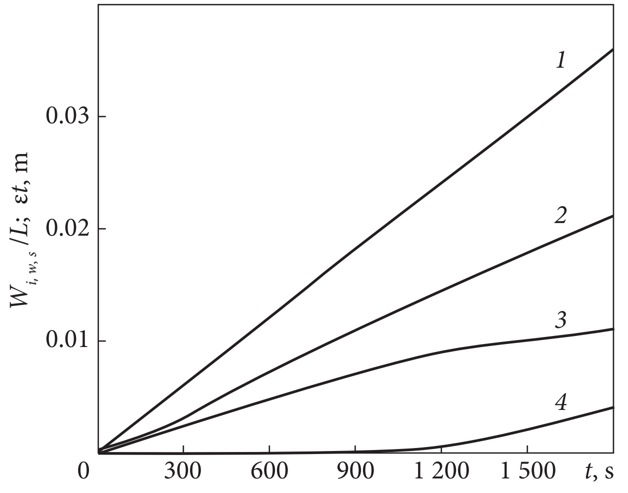

Fig. 3. Increase over time of specific reduced volumes of rainwater (εt), infiltrated into the soil (W_i/L), on the surface (W_w/L), flowing down the slope site (W_s/L): *1* — εt; *2* — W_i/L; *3* — W_w/L; *4* — W_s/L

there is a clear need for a detailed study of the intensive process of wetting soil that has been drained to varying degrees, using experimental methods in order to establish reliable values of z_w.

In the last series of examples, various components of rainfall were determined. The curves of increments of the total volumes of infiltration (W_i/L), accumulation on the surface (W_w/L), and water flowing down the slope (W_s/L) are shown in Fig. 3. To calculate them, (17) and (21) were used. Here, the consumption of prolonged (30 minutes) heavy rainfall on subsurface, surface runoffs, and the layer on the flooded surface is clearly presented. Due to the constancy of the rate ε (equal to $2\cdot10^{-5}$ m/s), the total volume of atmospheric recharge increases linearly according to graph 1. During the first five minutes, rainwater is evenly distributed between the aforementioned layer and the soil. However, due to the high permeability of the porous medium, rainwater is subsequently infiltrated into it. It should be noted that the calculations were performed exclusively at $z_w = 0$. It is obvious that the influence of capillary forces in accordance with Fig. 2 can significantly change the nature of rainfall distribution in favor of subsurface runoff. By the end of the calculation period, approximately one-third of the total rainfall volume is retained on the surface. Therefore, in order to adequately assess its final distribution between both runoff flows, it is advisable to carefully perform hydrological and filtration calculations for the second stage (after heavy rains) as well. In fact, a special methodology should be used, a detailed description of which is beyond the scope of this article due to its limited length.

Conclusions. Thus, due to changes in climatic conditions and, as a result, the nature of water exchange between the atmosphere and the earth, the development of the storm runoff theory has become relevant. An engineering method for calculating local and integral hydrological and filtration characteristics has been developed and tested, mainly for the stage of flooding of the sloping surface of a porous medium. The moisture condition of the soil and the capillary forces acting in the aeration zone deserve special attention due to their significant influence on rainwater infiltration. After the rainfall stops, surface and filtration flows continue for some time due to the water accumulated on the surface, with an expansion of the saturation zone (surface drainage phase).

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МОДЕЛЮВАННЯ ПІДЗЕМНОГО ТА ПОВЕРХНЕВОГО СТОКУ ПІД ЧАС СИЛЬНИХ ОПАДІВ НА СХИЛАХ

Сформульовано базові положення теорії зливових стоків і математичну задачу спільного поверхневого стоку та фільтрації дощової води на похилих сільськогосподарських землях у гідравлічній постановці. Отримано та обґрунтовано наближений розв'язок задачі. На його основі розроблено і проілюстровано метод інженерного розрахунку локальних та інтегральних гідрологічних, фільтраційних характеристик для першої стадії формування поверхневого та насиченого ґрунтового потоків (затоплення поверхні). Дано рекомендації щодо оцінювання обсягів продуктивної (ґрунтової вологи) і непродуктивної (що стікає назовні) води для першої і частково другої (осушення) стадій. Стосовно кліматичних і ґрунтово-гідрофізичних умов західного регіону України показано реальність істотного впливу капілярних сил на інфільтрацію і в цілому на водно-фізичну ситуацію на схиловій ділянці.

Ключові слова: *моделювання, інфільтрація, злива, схил, втрати напору, поверхневий стік, інженерний метод.*